Ph.D. QUALIFYING EXAM – Fall 2021

Description of Areas to be Examined

The fundamental principles of mathematics and the six basic areas of mechanical engineering to be covered by the examination and the minimum skills expected of the examinees in these areas are outlined below. While the list below is a fairly comprehensive guide, the exam may include any techniques found in a typical undergraduate Mechanical Engineering curriculum.

I. Mathematics

Suggested Texts:

1. Wylie, C.R. and Barrett, L. *Advanced Engineering Mathematics*, 6th Ed., McGraw-Hill, 1995
2. Boyce, W.E. and DiPrima, R.C. *Elementary Differential Equations and Boundary Value Problems*, 6th Ed., John Wiley, 1996
3. Chapra, C.C and Canale, R.P. *Numerical Methods for Engineers*, 4th Ed., McGraw-Hill, 2002

Specific List of Topics

1. ***Geometry and trigonometry***
    a. Plane geometry
    b. Trigonometric relations
    c. Calculation of dimensions on diagrams given some required information
2. ***Algebra (analytical and graphical)***
    a. Write systems of equations in matrix form
    b. Solve linear simultaneous equations
    c. Find roots of polynomials
    d. Fourier and Taylor series
    e. Dirac delta function
    f. Step function
3. ***Statistics and probability***
    a. Probability and cumulative density functions
    b. Variance, covariance, and standard deviation
4. ***Differentiation and integration (analytical and graphical)***
    a. Closed form derivatives and integrals
    b. Convolution integrals
    c. Power series
5. ***Ordinary differential equations (analytical and graphical)***
    a. Solve first and second order ODEs
    b. Convert second order ODEs to first order
    c. Laplace transforms
    d. Separation of variables
6. ***Numerical methods***
    a. Euler's method, trapezoidal method, Newton-Raphson method
    b. Root finding
    c. Integral calculation
    d. Numerical ODE solution
    e. Construct algorithms to implement methods
7. ***Linear algebra***
    a. Transposes
    b. Determinants
    c. Inverses
    d. Eigenvalues and eigenvectors
    e. Least-squares

II. Dynamics and Vibrations

Suggested Texts:

1. James, *Vibration of Mechanical and Structural Systems*, Harper, 1989
2. Rao, *Mechanical Vibrations*, Addison Wesley, 2nd ed., 1990
3. Inman, D. J., *Engineering Vibration*, 3rd ed., Prentice Hall, 2007

A. Fundamentals Principles and Skills

1. ***General***
    - a. Linear and non-linear differential equations.
    - b. Superposition principle.
2. ***Complex numbers and harmonic function***
    - a. Complex numbers
    - b. Complex harmonic function – Unit rotating complex function $e^{i\omega t}$
    - c. Definition of transfer function in the frequency domain
    - d. Prediction of transfer function from linear differential equations
3. ***Transforms***
    - a. Laplace Transform
    - b. Fourier Transform
    - c. Partial Fraction Expansion
4. ***1DOF Mechanical Systems***
    - a. Derivation of the equation of motion (EOM) (Newton and Lagrangian)
    - a. Derivation of free response of 1DOF systems (1st order and 2nd order systems).
    - b. Numerical integration of differential equations. Euler's method, recursive equation, and stability.
    - c. Forced response
        - o to arbitrary inputs
        - o to harmonic inputs
        - o Impulse response function
        - o Convolution
        - o Solution using Laplace Transform
        - o Steady-state solution in the frequency (complex) domain
        - o Frequency Response Function
        - o Transfer functions.
        - o Vibration isolation.
        - o Rotating unbalance
        - o Base excitation
5. ***Periodic Inputs, Spectrum and Decibel Scale***
    - a. Fourier series
    - b. Spectrum
    - c. Decibel scale
    - d. Steady-state response of a SDOF system to a periodic input.
6. ***MDOF mechanical systems***.
    - a. Derivation of EOM
    - b. Steady-state solution in the frequency (complex) domain
    - c. Dynamic Vibration Absorbers
    - d. Eigenvalue problem
    - e. Modal Analysis
    - f. Response MDOF systems using modal analysis

III. Machine Design

Suggested Texts:

1. R. Shigley, and L. Mitchell, *Mechanical Engineering Design*, 4th ed., McGraw-Hill, 1988.
2. R. Shigley, and C. Mischkey, *Mechanical Engineering Design*, 5th or 6th ed., McGraw-Hill.
3. Juvinall and Marshek, *Fundamentals of Machine Component Design*, 2nd ed., John Wiley, 1991.
4. R. G.Budynas and J. Keith Nisbett *Mechanical Engineering Design*, 8th ed., McGraw-Hill, 2007.

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Fundamental Principles and Skills

1. **Statics**. Be able to construct free-body diagrams and determine equilibrium forces and moments overall and on internal sections for stress calculations.
2. **Static Failure Theory**. Be able to design components with combined stress states for a prescribed or selected factor of safety or analyze existing components.
3. **Fatigue Failure Theory**. Be able to design or analyze components for fatigue failure.
4. **Stress Analysis**. Be able to calculate stresses in simple components like trusses, beams, thin-wall pressure vessels, shafts, etc.
5. **Design Practice**. Simplifying assumptions, stress concentration factors, factor of safety, common material behavior.

Sample Undergraduate Syllabus (Based on R.G. Budynas and J. Keith Nisbett *Mechanical Engineering Design*, 10th ed., McGraw-Hill, 2014. Does not include class handouts and other non-text materials.)

Topic	Reading	Suggested problems
Design, safety factor Materials: properties, processing, and selection	1-1 thru 1-3, 1-7 thru 1-12 2-1, 2-7, 2-12, 2-13, 2-15	2-1, 2-5
Equilibrium, FBD, shear and bending diagram Stress, strain, principal stresses, combined loading	3-1 thru 3-3 3-4 thru 3-8	3-8 3-15 (a and b), 3-20
Normal stress (axial, bending) Shear stress (transverse shear, torsion)	3-9 thru 3-10 3-11 thru 3-12	3-44 3-64
Combined stresses (2-plane bending), stress concentration Pressure vessels, press and shrink fits, contact stresses	3-12 thru 3-13 3-14, 3-16, 3-19	3-84 3-92, 3-96
Static failure theory: ductile (MSS)	5-1 thru 5-4	5.1 (a and e), 5.3 (a and e)
Static failure theory: ductile (DE, CM) Static failure theory: brittle (MNS, BCM, MM)	5-5 thru 5-7 5-8 thru 5-10	5.1 (c and d), 5.10, 5.12 5.21, 5.25
Buckling: Euler, Johnson; buckling design FEA introduction and fracture Introduction	4-11 thru 4-13 5-12	4-104, 4-105, 4-106 5-84, 5-85
Fatigue introduction Fatigue strength estimation	6-1 thru 6-4 6-7 thru 6-8	6-5 6-3
Fatigue stress concentration Fluctuating stresses, fatigue failure theory	6-9 thru 6-10 6-11 thru 6-12	6-16 6-27(a)
Fluctuating torsion, combined loading Overload introduction and formulation	6-13 thru 6-14 6-16 thru 6-17	6-3
Cumulative fatigue	16-5	6-59, 6-60
Thread standards, power screws, stiffness model Bolt strength, preload	8-1 thru 8-5 8-6 thru 8-8	8-1, 8-4, 8-11 8-26
Bolted joint in tension static design Bolted joint in tension fatigue design, shear design	8-9 thru 8-10 8-11 thru 8-12	
Welds introduction, butt and fillet welds Welded joint stress: torsion, bending	9-1 thru 9-2 9-3 thru 9-4	
Weld strength	9-5 thru 9-7	

IV. Control Theory

Suggested Texts:

1. Dorf and Bishop, *Modern Control Systems*, 11th Ed., Prentice-Hall, 2008.
2. Ogata, *Modern Control Engineering*, 5th Ed., Prentice-Hall, 2009

Any textbook that presents an introduction into the analysis and design of control systems is a suitable study guide.

Fundamental Principles and Skills

1. **Modeling**. Be able to develop up to 5th order ordinary differential equation models for simple mechanical, electrical, and electro-mechanical systems.
2. **Open and Closed Loop System Analysis**. Be able to determine stability and characteristics of the response of open and closed loop systems using Routh-Huritz, Bode, or Root-Locus techniques. Be able to calculate gain and phase margins. For second-order systems, be able to determine settling time, rise time, percent overshoot, damping ratio, and natural frequency.
3. **Control System Design**. Be able to develop controllers to achieve performance specifications. Be able to design PID and lead-lag controllers to meet performance specifications.

Sample Undergraduate Syllabus (Based on Dorf and Bishop, *Modern Control Systems*, 11th Ed., Prentice Hall, 2008. Does not include class handouts and other non-text materials.)

Topic	Reading	Problems
Introduction to Course and Overview Linearization and ODE's	Preface, 1.1 -1.12 2.1-2.3	
Laplace, pole-zeros Transfer funct. and block diagrams Block Dia. and Examples:	2.4 2.5-2.6 2.6	P1.15, E2.2, E2,17, P2.3, P2.9 and P2.51
MATLAB, Simulation Final Value, Performance, sensitivity Open-loop vs closed loop, Error	2.6, 2.10 4.1 -4.2 4.3, 4.5	E2.23, P2.51 (in matlab), MP2.6, E4.1 and E4.7 (a&b)
Disturbance Performance of 2nd Order sys. Performance of 2nd Order sys.	4.4 5.1, 5.3 5.3	E4.7 (c&d), E4.9, MP4.3, P5.4 and AP5.4
Test Signals, Extra pole/zero	5.2, 5.4, 5.6	P5.2 and AP5.5
Steady State Error Stability, Root locus	5.7 -5.14 6.1, 7.1 -7.2	P6.7 and MP6.2
Root Locus Root Locus Root Locus	7.3 7.3 7.3	E7.7, E7.19, P7.1, P7.6 and AP7.9
PID Controllers: DP Given PID Controllers PID Controllers	7.7 7.7 7.8 -7.12	P7.1 for K<0, P.7.19, DP7.13, MP7.1, MP7.6
Frequency Response Frequency Response Bode Plots	8.1-8.2 8.2 8.3	E 8.3 and P 8.2 a,b,c
Bode Plots Bode Plots Bode Plots	8.3	P8.2 and P8.23
Measurement and Specs Controllers Gain and Phase margins	8.4, 8.5 10.16	P8.15, P8.24, AP 8.2, MP8.7 and MP8.9

Nyquist Criterion	9.3, 9.4, 9.10	
Nyquist Criterion Lead Controllers	9.3, 9.4, 9.10 10.1 – 10.3	P9.2, P9.4, P9.16 and AP9.4
Lead Compensator Design Lag Control Lead-Lag Control	10.4 – 10.6 10.7-10.8 10.9-10.16	SD1a and SD1b

V. Heat Transfer

Suggested Text:

1. Bergman and Lavine., *Fund. of Heat and Mass Transfer*, 8th ed., John Wiley, 2017.

Fundamental Principles and Skills

1. **Energy Balance**. Be able to derive the "energy equation" on the basis of an energy balance on both finite and differential volume elements (solid or fluid). The derivation should include the possibility of internal heat generation (sources). Be able to obtain analytical solutions for the case of steady conditions in solids and liquids with convection conduction and radiation..
2. **Extended Surfaces**. Be able to derive the ordinary differential equation governing the temperature distribution in an extended surface (fin) and, for the case of a constant cross-section, be able to solve it subject to classical boundary conditions (insulated tip, infinitely long, known tip temperature).
3. **Lumped Heat Capacity**. Know and be able to apply the criterion for using the lumped heat capacity approach to solving transient problems, and be able to formulate and solve multi-domain problems using this approach.
4. **Radiation Heat Transfer**. Be able to set up and solve the equations governing radiation heat transfer among diffuse, gray surfaces. This includes the ability to evaluate the needed radiation view factors for simple geometries.
5. **Convection Heat Transfer**. Be familiar with the origins of and the role played by non-dimensional groups in local and average convection correlations. Be able to solve for heat transfer for constant-property flow across bluff bodies and for fully-developed internal flow heat transfer using correlations available in typical undergraduate texts.
6. **Conduction Heat Transfer**. Be able to derive and know how to solve the explicit form of the finite-difference equations for unsteady, multidimensional conduction in rectangular coordinates, and be able to discuss system stability.

Sample Undergraduate Syllabus (Based on Bergman and Lavine, *Fund. of Heat and Mass Transfer*, 8th ed., John Wiley, 2017. Does not include class handouts and other non-text materials.)

Period	**Topic**	**Reading Assignment**	**Problems Due**
1	Control Volumes		
2	Energy balances	1.1-1.7, 2.1-2.5	1.12, 1.21, 1.35
3	Plane and Radial geometry	3.1-3.4	1.51, 1.61, 2.6
4	Internal generation	3.5	3.5ab, 3.7, 3.49
5	Series/Parallel Paths		3.70, 3.77, S1Lab
6	Fins	3.6	3.11, 3.48ab, 3.65
7	Fin Arrays	4.1, 4.3	3.102, 3.107
8	Fins		3.111, S2Lab
9	Finite-difference method	4.4-4.6	3.105, 3.121, 3.124
10	Transient, Lumped capacity method	5.1-5.3	4.35, 4.42
11	Solution methods		4.45, 5.9, S3Lab
12	Transient solutions	5.3	4.52, 4.56a, 5.10
13	Semi-infinite solid	5.7	5.17, 5.19, 5.67
14	**Test Review**		5.25, 5.72a, S4Lab
15	**Test 1**		
16	External convection	6.1-6.3, 6.6-6.8	
17	Boundary Layers		6.5, 6.24
18	External convection	7.1-7.4	6.7, 6.8, 6.40
19	External convection	7.5, 7.9	7.13, 7.19
20	Integrating for average h		7.21, 7.34a, S5Lab

21	Internal flow basics	8.1-8.2	7.8, 7.55
22	Internal flow basics	8.3-8.5	7.32, 8.3, 8.13
23	Enthalpy balances		8.7a, 8.21, S6Lab
24	Internal flow applications	8.6, 8.10	7.65ab, 8.28a, 8.43a
25	Internal flow applications		8.30a, 8.27, 8.57
26	**h estimation**		8.64, S7
27			8.54, 8.50ab, 8.70
28			
29	Test Review		
30	**Test 2**		
31	Intro to heat exchangers – LMTD	11.1-11.3	
32	LMTD and MTD		11.2a, 11.15
33	Effectiveness-NTU analysis	11.4	11.1, 11.20, 11.23ab,
34	Effectiveness-NTU analysis	11.5, 11.7	11.11, 11.49, 11.50
35	Effectiveness-NTU analysis		11.12, 11.34, S9
36	Radiation shape factors	13.1	11.62, 11.63a, 11.64
37	Radiation exchange – black surfaces	13.2	12.17, 12.13a, 13.1
38	Energy balances		13.3, 13.15, S10
39	Radiation exchange – gray surfaces	13.3	13.4ab, 13.8, 13.17
40	Radiation exchange - enclosures	13.3	13.33, 13.25, 13.38
41	**Gray surfaces**		13.42abc, 13.43, S11Lab
42	Multimode Exchange	13.4	13.62ab, S12
43	Radiation problem review		13.63, 13.73ab, 13.77ab

VI. Fluid Mechanics

Suggested Text:

1. Munson, Young, and Okishi, *Fundamentals of Fluid Mechanics*, 5th ed., John Wiley and Sons, 2006.
2. R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics,* 4th ed., Wiley & Sons, 1992.
3. F. M. White, *Fluid Mechanics*, 3nd ed., McGraw-Hill, 1994.

Fundamental Principles and Skills

1. **Hydrostatics**. Be able to compute pressures within static fluid systems and force distributions on bounding surfaces.
2. **Conservation Principles**. Be able to apply the principles of conservation of mass, momentum, and energy to differential and finite volume elements to obtain the equations of continuity and motion.
3. **Incompressible Ideal Fluids**. Be able to solve one-dimensional steady flow (hydrodynamics) problems using Euler's and Bernoulli's equations.
4. **Incompressible Viscous Flow**. Be able to describe and discuss the concepts of laminar and turbulent boundary layers and the criterion for transition to turbulence in exterior and interior flows. Be able to use simple boundary layer models to compute frictional drag. Also, be able to compute the pressure drop for fully-developed laminar flow in pipes. Finally, be able to use drag coefficients and friction factors to compute drag and pressure drop for flow over bluff bodies and within pipes.

Specific List of Topics

1. ***General***
   a. Properties of fluids: density, viscosity, and bulk modulus
   b. Concept of Newtonian and non-Newtonian fluids
2. ***Hydrostatics***
   a. Concept of pressure and hydrostatic law
   b. Manometers; gage vs. absolute pressure
   c. Force acting on planar surfaces immersed in static fluids: magnitude and action center of the effective force
   d. Buoyancy
3. ***Elementary dynamics***
   a. Lagrangian and Eulerian description of fluid motion
   b. Classification of fluid flows: 1D, 2D, 3D, and steady flows
   c. Acceleration of fluid particles: local and convective contributions
   d. Differential form of the Bernoulli equation and its physical interpretation
   e. Algebra form of the Bernoulli equation and its physical interpretation
   f. Concept of static, dynamic, stagnation, and total pressures and their measurement
   g. Application of the Bernoulli equation for steady, inviscid flows
4. ***Control volume analysis and conservation laws***
   a. Reynolds transport theorem and its applications in control volume analysis
   b. Mass conservation in a control volume
   c. Linear momentum conservation in a control volume
   d. Energy conservation in a control volume, especially for one-dimensional, steady, and uniform flows

5. ***Dimensional analysis***
    a. PI-theorem
    b. Method of constructing dimensionless groups
    c. Modeling based on dimensional analysis; perfect and distorted models
6. ***Internal flows***
    a. Concept of entrance region/length
    b. Concept of turbulent vs. laminar flows, empirical criteria for turbulent flow in round pipes
    c. Hydraulic diameter
    d. Dimensional analysis applied to internal flows: major loss (including Moody chart) and minor loss
    e. Pipe flow analysis
7. ***Differential analysis***
    a. Continuity equation: general equation and the simplified equation for incompressible flows
    b. Kinetics of fluid motion, stress in Newtonian fluids
    c. Momentum equations: Euler and Navier Stokes equations. Emphasis is on the physical interpretation of the terms in the equations
    d. Exact solution of unidirectional laminar flows, e.g., Couette and pressure driven flows
    e. Concept of turbulent stress
8. ***External flows***
    a. Concept of external flows
    b. Drag and lift forces: definitions and origins
    c. Characteristics of external flow and its dependence on the Reynolds number
    d. Concept of boundary layers, in particular the evolution of the boundary layer along external surfaces (boundary layer thickness and wall stress)
    e. Concept of streamlined objects and blunt bodies and the characteristics of flow near them
    f. Drag force calculation: drag coefficient and its dependence on flow (e.g., the Reynolds number) and object shape; pressure drag vs. viscous drag
    g. Lift force calculation: lift coefficient and its dependence on flow and object shape

VII. Thermodynamics

Suggested Text:

1. Van Wylen and Sonntag, *Fundamentals of Classical Thermodynamics*, English / SI Version 3rd ed., John Wiley, 1986.
2. Cengel and Boles, *Thermodynamics – an Engineering Approach*, 5th ed, McGraw-Hill, 2006.
3. Moran and Shapiro, *Fundamentals of Engineering Thermodynamics*, 5th ed., Wiley, 2004.

Fundamental Principles and Skills

1. **<u>The First Law</u>**. Be able to apply the first law of thermodynamics to various closed systems and open systems for both steady flow and transient cases. Be able to handle situations involving ideal gases, ideal gas mixtures, solids, liquids, phase change, and/or chemical reactions.
2. **<u>The Second Law</u>**. Be able to apply the second law of thermodynamics various closed systems and open systems for both steady flow and transient cases. Be able to handle situations involving ideal gases, ideal gas mixtures, solids, liquids, phase change, and/or chemical reactions. Be able to calculate entropy changes and to determine whether specified processes are reversible, irreversible, or even possible. Be able to use the second law to determine the states, processes, and energy quantities that correspond to reversible devices.
3. **<u>Physical Property Relationships</u>**. Be able to either calculate or determine from tables of thermodynamic properties the quantities needed to work with the first and second Laws. This includes knowing how to work with ideal gases, mixtures of ideal gases, liquids, solids, and substances which change phase.

Specific List of Topics

1. Properties
   a. Use of tables to find properties of pure, simple compressible substances and gases
      i. Pressure, temperature, specific internal energy, specific volume, specific enthalpy, specific entropy
      ii. Single phase, two-phase (quality), and three-phase (mass fraction)
      iii. Compressed liquid and solid approximation
   b. Phase diagrams - plotting on *P*-*v* and *T*-*v* diagrams to identify the phase
   c. Ideal gas model, equation of state, properties as a function of temperature or temperature and pressure
   d. Specific heats at constant volume and constant pressure (constant and temperature varying) to calculate specific internal energy and specific enthalpy differences
   e. Entropy difference from *Tds* equations (i.e., from the Gibbs relation) including the use of gas tables and the specific heats at constant volume and constant pressure

2. Closed Systems
   a. Energy balance (consequence of the 1st Law)
      i. Change in energy of the system comprised of internal energy, kinetic energy, potential energy changes
      ii. Energy transfer due to work and heat interactions; sign convention for work (positive out) and heat (positive in) interactions
   b. Work calculation from a variety of sources
      i. Boundary, electrical, and shaft work
   c. Multiple phases in a system (e.g., gas and solid)
   d. Being able to sketch processes / cycles on a *P*-*v* diagram

3. Control Volume (Open) Systems
   a. Conservation of mass (mass balance)
      i. Application to one inlet/one exit, multiple inlets/multiple exits
   b. Energy balance on a system (consequence of the 1st Law)
      i. Application to one inlet/one exit, multiple inlets/multiple exits

   c. Steady-state analysis applying mass and energy balances to the control volume for various components
      i. Nozzles, diffusers, turbines, compressors, pumps, heat exchangers, throttling devices
      ii. Determination of the work, heat transfer, and/or conditions at inlets or exits
   d. Transient analysis applying mass and energy balances to the control volume of systems
      i. Leak problems, tank filling / exhausting, etc.
      ii. Determination of the work, heat transfer, the mass in control volume, and/or conditions at an inlet or exit

4. ***Entropy balances for Closed Systems***
   a. Determination if a process is possible
   b. Isentropic processes
   c. Use of energy and entropy balances to evaluate systems
5. ***Entropy balances for Control Volume Systems***
   a. Determination if a process is possible
   b. Isentropic processes
   c. Steady-state analysis on components using mass, energy, and entropy balances
      i. Isentropic and actual processes
      ii. Nozzles, diffusers, turbines, compressors, pumps, heat exchangers, throttling devices
      iii. Component efficiencies
   d. Transient analysis using mass, energy, and entropy balances
6. ***Vapor Power Cycles***
   a. Rankine cycle
   b. Improvement in performance through superheat, sub-cooling, reheat, and regeneration
   c. Calculation of the work and heat transfer for processes and entire cycles
   d. Being able to sketch ideal and actual cycles on *T-s* and *P-v* diagrams
   e. Isentropic and actual processes, cycle efficiency, maximum cycle efficiency
7. ***Air Power Cycles***
   a. Models for internal combustion engines with the combustion gases modeled as air
      i. Otto, Diesel, and dual cycles; air standard and cold-air standard analyses
      ii. Being able to sketch ideal and actual cycles on *P-v* and *T-s* diagrams
      iii. Calculation of the work and heat transfer for processes and entire cycles
      iv. Isentropic and actual processes, cycle efficiency, maximum cycle efficiency
      v. Calculation of other parameters to describe performance such as the mean effective pressure, compression ratio, cutoff ratio
   b. Models for gas turbine cycles with the combustion gases modeled as air
      i. Brayton cycle; air standard and cold-air standard analyses
      ii. Cycles with improved Brayton cycle performance through regeneration, reheat, and intercooling
      iii. Being able to sketch ideal and actual cycles on *P-v* and *T-s* diagrams
      iv. Calculation of the work and heat transfer for processes and entire cycles
      v. Isentropic and actual processes, cycle efficiency, maximum cycle efficiency
      vi. Calculation of other parameters to describe performance such as the back work ratio
8. ***Refrigeration and Heat Pump Vapor Compression Cycles***
   a. Ideal and actual cycles
   b. Being able to sketch ideal and actual cycles on *P-v* and *T-s* diagrams
   c. Calculation of the work and heat transfer, including refrigeration capacity, for processes and entire cycles
   d. Isentropic and actual processes, cycle efficiency, maximum cycle efficiency
9. ***Mixture***s
   a. Describing mixtures using moles, mass, molecular weight, mass fractions, mole fractions
   b. Dalton’s model (partial pressures) and Amagat’s model (partial volumes)

- c. Properties for mixtures (specific internal energy, specific enthalpy, specific entropy, specific heats at constant volume and pressure, etc.)
- d. Analysis of closed systems containing a mixture using energy and entropy balances
- e. Analysis of steady-state control volume systems with mixtures using mass, energy, and entropy balances
- f. Analysis of transient control volume systems with mixtures using mass, energy, and entropy balances
- g. Psychrometrics (air – water vapor) systems
    - i. Humidity ratio, relative humidity, mixture specific enthalpy, mixture specific entropy, dew point temperature, condensation, wet bulb temperature, dry bulb temerpature, psychrometric chart
    - ii. Mass and energy balances on systems with condensate, constant moist air composition, dehumidification, humidification

10. Combustion

- a. Chemical reaction balances
    - i. Stoichiometric air, excess air, deficient air, air-fuel ratio, percent theoretical air, equivalence ratio
- b. Quantifying the specific enthalpy and the enthalpy of formation
- c. Energy balance analysis of closed and control volume (open) systems
    - i. Products of combustion with water as vapor or liquid
- d. Enthalpy of reaction and the lower and higher heating values

Appendix A

The Energy Equation, Boundary Layer, Differential Operator and Navier-Stokes Equation were obtained from the Munson's textbook.

IV. Energy Equation

By applying Reynolds transport theorem to 1st Law of Thermo

$$\left(\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}\right) \text{ for system}$$

Let B = energy E

$$\beta = \frac{dE}{dm} = e = \underset{\substack{\text{internal}\\\text{energy}}}{u} + \underset{\text{KE}}{\tfrac{1}{2}V^2} + \underset{\text{PE}}{gz}$$

Reynolds transport eq.:

$$\frac{d}{dt}(B_{sys}) = \frac{d}{dt}\left(\int_{cv} \beta\rho d\,Vol\right) + \int_{cs} \beta\rho(\rho\vec{v}\cdot\vec{n})\,dA$$

=> 1st Law for cv

$$\underset{\substack{\text{heat added}\\\text{to system}}}{\frac{dQ}{dt}} - \underset{\substack{\text{WD}\\\text{by system}}}{\frac{dW}{dt}} = \frac{dE}{dt} = \frac{d}{dt}\left[\int_{cv} e\rho d\,(Vol)\right] + \int_{cs} \underbrace{e\rho(\vec{v}\cdot\vec{n})\,dA}_{\dot{m}}$$

If assume:
1. no heat transfer
2. steady
3. 1-D inlet and outlet ($\dot{m} = \dot{m}_{out} = \dot{m}_{in}$)
4. incompressible

$$-\dot{w} = \dot{m}e_{out} - \dot{m}e_{in}$$

divided by $\dot{m}$ $\quad -w = e_2 - e_1 = (u + \tfrac{1}{2}v^2 + gz)_2 - (u + \tfrac{1}{2}v^2 + gz)_1$

Consider w, 2 components:

a. Shaft work (w_s) - work done by a machine (pump, compressor, piston, etc.) protruding thru cs to cv

b. Pressure work (w_p) - due to pressure forces at cs, can be accounted for by replacing internal energy (u) with enthalpy (h) on RHS of eq.

i.e. $h = u + P/\rho$

Steady-flow energy equation (energy per unit mass)

$$-w = (u_2 + \frac{1}{2}v_2^2 + gz_2) - (u_1 + \frac{1}{2}v_1^2 + gz_1)$$

$$-w_s = (h_2 + \frac{1}{2}v_2^2 + gz_2) - (h_1 + \frac{1}{2}v_1^2 + gz_1)$$

$$-w_s = \left(\frac{p_2}{\rho} + u_2 + \frac{1}{2}v_2^2 + gz_2\right) - \left(\frac{P_1}{\rho} + u_1 + \frac{1}{2}v_1^2 + gz_1\right)$$

divided by g

$$-\frac{w_s}{g} = \left[\frac{p_2}{\rho g} + \frac{u_2}{g} + \frac{v_2^2}{2g} + z_2\right] - \left[\frac{p_1}{\rho g} + \frac{u_1}{g} + \frac{v_1^2}{2g} + z_1\right]$$

(unit: length or head)

or

$$\frac{p_1}{\rho g} + \frac{u_1}{g} + \frac{v_2^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{u_2}{g} + \frac{v_2^2}{2g} + z_2\right] + \frac{w_s}{g}$$

where w_s/g is a measure of energy/(mass of fluid) transferred to the fluid due to shaft work (negative for a pump, positive for a turbine). We will refer to this as the head change due to shaft work, h_s (given in ft or in).

The equation can be rearranged

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2\right] + \underbrace{\frac{w_s}{g}}_{h_s} + \frac{u_2 - u_1}{g}$$

for steady, incompressible flow with friction, $(u_2 - u_1) > 0$, is a measure of loss of available energy due to friction losses. We will define a fricitonal head loss $h_f \equiv (u_2 - u_1)/g$, so that the energy equation becomes

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \left[\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2\right] + h_s + h_f}$$

Note: h_s - for pump, + for turbine.

II. Boundary-layer flows

A. Laminar flat-plate boundary layer

Exact solution for equation of motion + continuity (Blasius sol'n) neglect g, 2D, steady flow, zero pressure gradient (dp/dx = 0), incompressible

Continuity: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$

$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ——————(1)

Momentum (Navier-Stokes eq.)

$$x:\ \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \text{ ——————(2)}$$

BC at $y = 0$, $u = 0$

at $y = \infty$, $u = U$

•
•
•

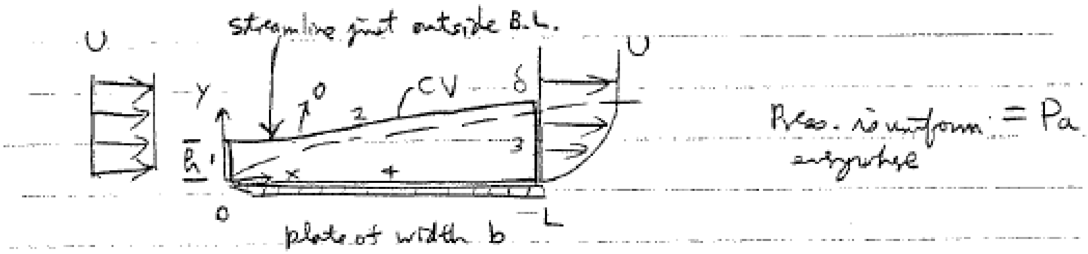
$$\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$$

wall shear stress coe, $c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}}$

II. Boundary-layer flows - continued

B. Momentum-integral estimates for BL

same assumptions as before, except for both laminar & turb. BL, instead of exact solution, use cv analysis



Momentum Eq. in x-dir (nonuniform vel. profile)

$$\Sigma F_x = \cancel{\frac{d}{dt}(\ \ )}^{0} + \int u_x\, \rho(\vec{v}\cdot\vec{n})\, dA$$

$$-F_D = \rho \int_1 u_x(\vec{v}\cdot\vec{n})\, dA + \rho \int_3 u_x(\vec{v}\cdot\vec{n})\, dA + \cancel{\int_2}^{0} + \cancel{\int_4}^{0}$$

$$= \rho \int_0^h U\,(-U)\, b\,dy + \rho \int_0^\delta u(+u)\, b\,dy$$

$$F_D = \rho U^2 bh - \rho b \int_0^\delta u^2\, dy$$

$h = ?$

Continuity (nonuniform vel. profile)

$$\cancel{\int_{cv} \frac{d\rho}{dt}}^{0}\, d\,\text{Vol} + \int_{cs} \rho\,(\vec{v}\cdot\vec{n})\, dA = 0$$

$$\rho \int_0^h (-U)\, b\,dy + \rho \int_0^\delta u\,b\,dy = 0$$

$$\Rightarrow Uh = \int_0^{\delta} u\,dy$$

$$\therefore\ F_D = \rho b U \int_0^{\delta} u\,dy - \rho b \int_0^{\delta} u^2\,dy$$

$$F_D = \rho b \int_0^{\delta} u(U-u)\,dy$$

$$F_D(x) = \rho b \int_0^{\delta(x)} u(U-u)\,dy$$

Note $\rho U^2 \theta \equiv \int_0^{\delta} \rho u(U-u)\,dy$ where θ = momentum thickness

$$\therefore\ F_D(x) = \rho b U^2 \theta \Rightarrow \frac{dF_D}{dx} = \rho b U^2 \frac{d\theta}{dx} \qquad \text{———— (1)}$$

However $F_D = b \int_0^{x} \tau_w(x)\,dx$ or $\frac{dF_D}{dx} = b\tau_w$ ———— (2)

Equating (1) and (2)

$$\Rightarrow \tau_w = \rho U^2 \frac{d\theta}{dx} \text{ ————(3)}$$ momentum-integral relation for both laminar or turbulent flow

For laminar flow, assume $u(y) = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)$,then evaluate $\theta = fn(\delta)$

$$\tau_w \equiv \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = fn(\delta)$$

⋮

sub θ, τ_w into (3) => diff. eq. => solving

Integral analysis	vs	Exact solution
$\frac{\delta}{x} \approx \frac{5.5}{\sqrt{Re_x}}$		$\frac{5.0}{\sqrt{Re_x}}$
	compare to	
$c_f \approx \frac{0.73}{\sqrt{Re_x}}$		$\frac{0.664}{\sqrt{Re_x}}$

SOME CONVENIENT DIFFERENTIAL OPERATORS IN FLUID MECHANICS

I. Total acceleration (substantial)

$$\underbrace{\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t}}_{\text{local acc.}} + \underbrace{\left[u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}\right]}_{\text{convective acc.}}$$

$$\text{vector-gradient operator } \nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\text{so that } \vec{V}\cdot\nabla = (u\hat{i} + v\hat{j} + w\hat{k})\cdot\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\text{and } (\vec{V}\cdot\nabla)\vec{V} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \quad \text{(convective acc.)}$$

$$\boxed{\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}}$$

local convective

II. Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\nabla\cdot(\rho\vec{V}) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot(\rho u\hat{i} + \rho v\hat{j} + \rho w\hat{k}) = \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0}$$

III. Momentum Equations

a. Euler's eq. (inviscid flow)

$$x:\ \rho g_x - \frac{\partial P}{\partial x} = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$

$$y:\ \rho g_y - \frac{\partial P}{\partial y} = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right]$$

$$z:\ \rho g_z - \frac{\partial P}{\partial z} = \rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right]$$

$$\nabla P = \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

$$\vec{g} = \hat{i}g_x + \hat{j}g_y + \hat{k}g_z$$

$$\boxed{\rho\vec{g} - \nabla P = \rho\frac{D\vec{V}}{Dt}}$$

b. Navier-Stokes eq.

$$x:\ \rho g_x - \frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]$$

$$y:\ \rho g_y - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right]$$

$$z:\ \rho g_z - \frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] = \rho\left[\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right]$$

$$\rho\vec{g} - \nabla P + \quad ? \quad = \rho\frac{D\vec{V}}{Dt}$$

$$\nabla^2 = \nabla\cdot\nabla = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\cdot\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{(Laplacian operator)}$$

$$\Rightarrow \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\Rightarrow \quad ? \quad = \mu\nabla^2\vec{V}$$

=>

$$\boxed{\rho\vec{g} - \nabla P + \mu\nabla^2\vec{V} = \rho\frac{D\vec{V}}{Dt}}$$