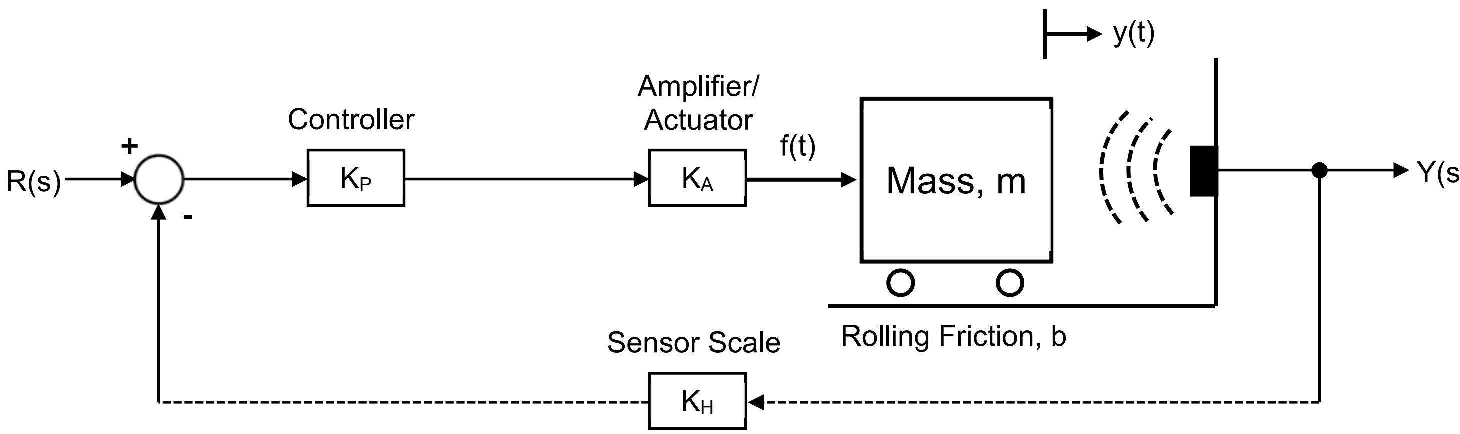
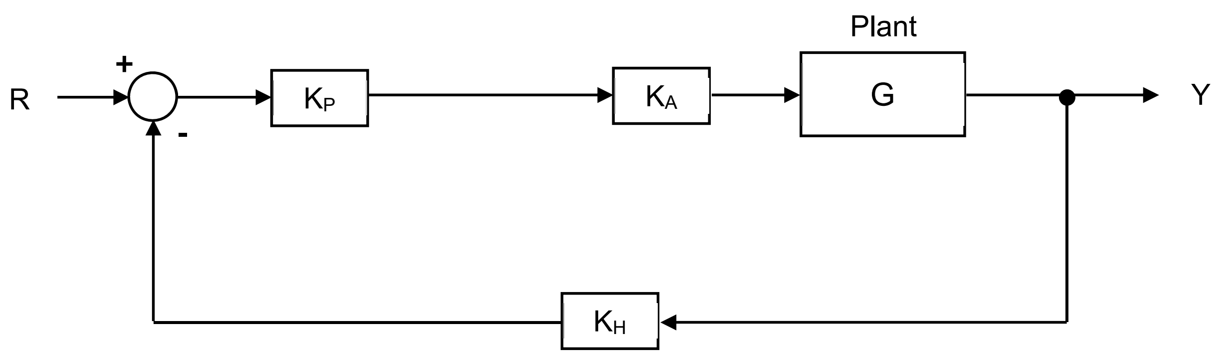
Controls - 2

This systems analysis problem pertains to the system block diagram below. There is a controller that consists of a proportional constant K_P, a power amplifier constant K_A, a mechanical system consisting of a rolling cart (mass, m), rolling viscous friction (damping, b) and a retro-reflective position sensor (optical or sonic). The scale of the position sensor is represented by K_H in the feedback loop.



a. Write the equation of motion for the rolling mass in terms of input force f(t) and output displacement y(t), where y(t) = 0 corresponds to the initial starting position of the mass. **(4 points)**

b. Find the expression for output displacement with respect to input force, Y(s)/F(s), in the frequency domain (the s plane). This is your plant, G, in the diagram below. **(8 points)**



c. Using your function for G, find the closed-loop transfer function, T_1, for the system above. **(5 points)**

d. If input R = A/s (step input), find the value of K_H in terms of A for a steady-state displacement of $y_{SS} = 0.2$ meters. **(5 points)**

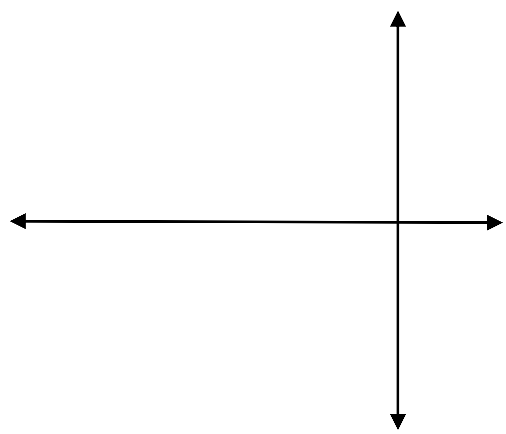
e. If $K_P = 1$, $K_A = 1$, $K_H = 1$, m = ½ kg, and b = 1 Ns/m, write the closed-loop transfer function, T_1, in standard form. **(4 points)**

f. Find the roots of the characteristic equation of T_1. **(4 points)**

g. Find the values for the damping ratio (ξ) and the undamped natural frequency (Wn) for T_1. **(4 points)**

ξ =
Wn =

h. Draw in your solution the pole-zero plot for T_1, and include/label as many parameters as possible. **(6 points)**

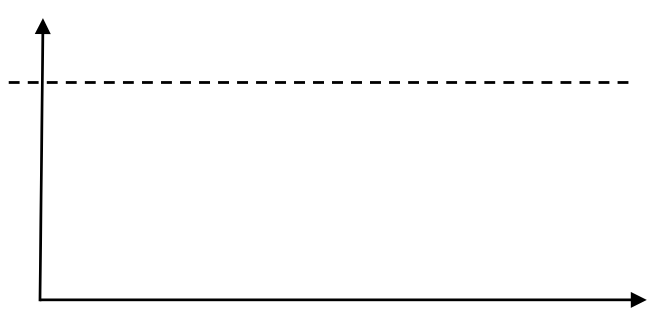


i. For T_1, find the expression for y(t) when r(t) = u(t), the unit step function. **(2 points)**

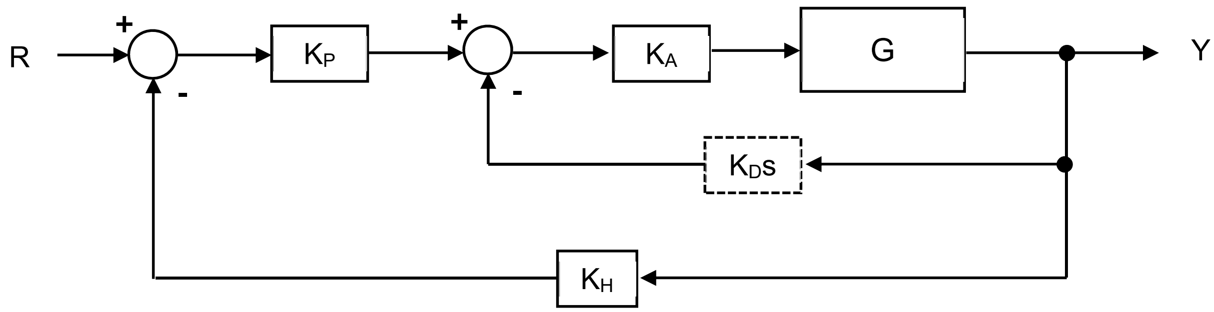
j. For T_1, estimate the percent overshoot, time-to-peak, and 2% settling time for a step input (r(t) = u(t)). **(6 points)**

PO =
Tp =
Ts =

k. Sketch in your solution the response to a unit step input (1/s) to T_1, and include/label as many parameters as possible. **(5 points)**



l. Now add another negative feedback loop that differentiates the output y and multiplies the result by a constant, K_D, as shown below. For your same plant, G, from above, if $K_P = 2$, $K_A = 1$, $K_H = 1$, and $K_D = 1$, write the closed-loop transfer function, T_2, in standard form. **(10 points)**

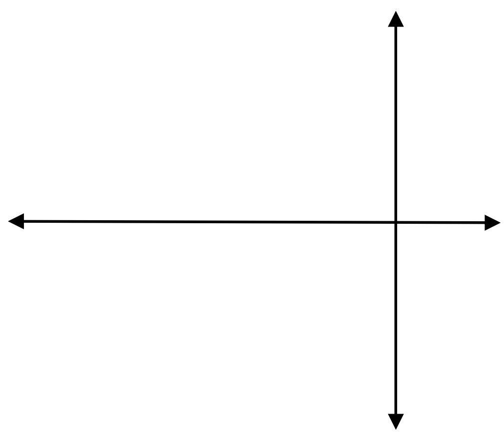


m. Find the roots of the characteristic equation of T_2. **(4 points)**

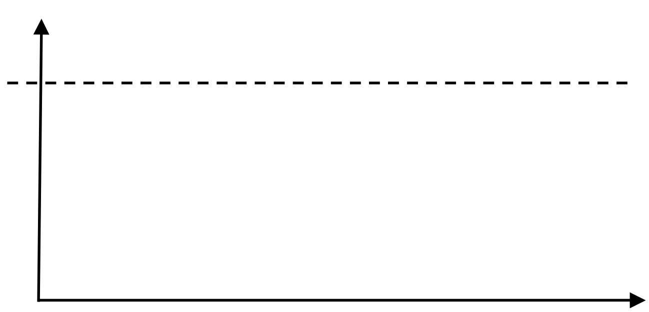
n. Find the values for the damping ratio (ξ) and the undamped natural frequency (Wn) for T_2. **(4 points)**

ξ =
Wn =

o. Draw in your solution the pole-zero plot for T_2, and include/label as many parameters as possible. **(4 points)**



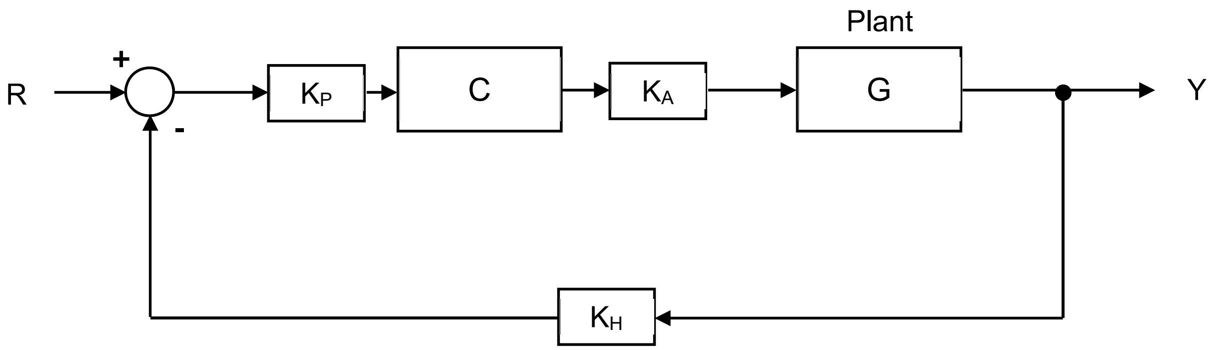
p. Sketch in your solution the response to a unit step input (1/s) to T_2, and include/label as many parameters as possible. **(4 points)**



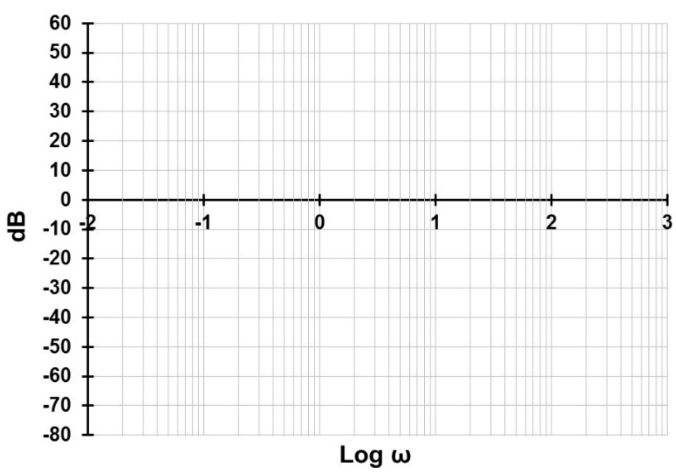
q. Compare/contrast T_1 and T_2. **(4 points)**

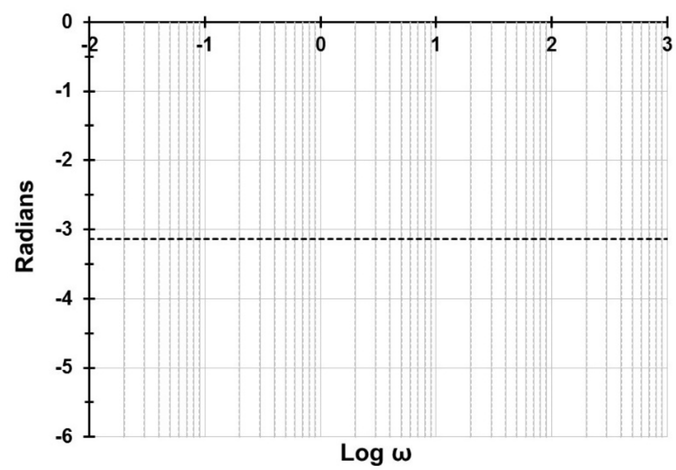
r. A new dynamic control element is added to the original system, as shown below. As before, $K_A = 1$, $K_H = 1$, and within the plant, m = ½ kg, and b = 1 Ns/m. But now, $K_P = 8.4$. The control element has the form:

$$C = \frac{2\,(s + 8)}{8\,(s + 2)}$$



Investigate the relative stability of the system by sketching in your solution the appropriate Bode amplitude and phase responses. Describe the system's stability. Support your answer using characteristics of the Bode plots. **(13 points)**





s. What type of controller is C? **(2 points)**

t. For a step input, determine the steady state error for the control system shown in part r. Support your answer. **(4 points)**