

Controls - 1

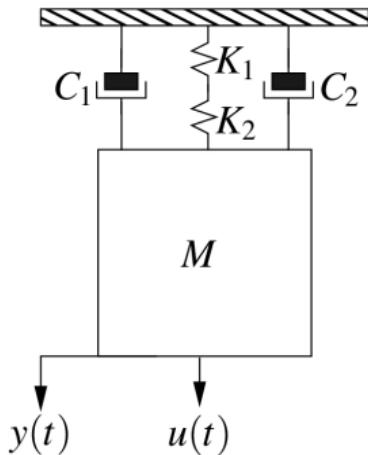
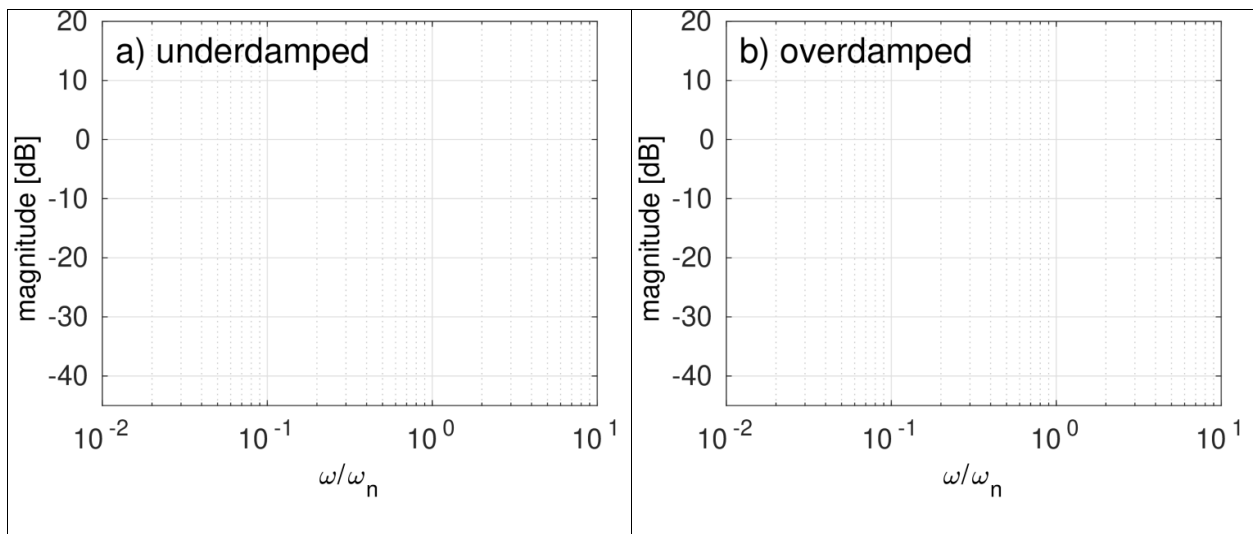


Figure 1: Mechanical system with input force $u(t)$ and output displacement $y(t)$. The system parameters are the mass M , the spring constants K_1, K_2 and the damping coefficients C_1, C_2 .

1. **(15 points)** Write down the differential equation that describes the mechanical system shown in Figure 1 below.
2. **(25 points)** Provide qualitative sketches of the magnitude portion of the Bode plots of the mechanical system's transfer functions assuming that the system parameters are chosen to create:
 - (a) an underdamped system and
 - (b) an overdamped system.

Use the templates provided in Figure 2 and insert one sketch of the respective transfer function (underdamped or overdamped) into each template. For each of the two graphs, include the low- and high-frequency asymptotes and write the values of the slope for each of the asymptotes next to the respective asymptote into the graph. Assume that the DC gain of the transfer function between the input u and the output y is 1.



Problems

Figure 2: Magnitude portion of the Bode plots for the transfer functions of the mechanical system (Figure 1) assuming that the system parameters are chosen to create (a) an underdamped system and (b) an overdamped system

3. **(30 points)** The signals shown in Figure 3 are candidates for impulse responses of the mechanical system in Figure 1. For each of these signals, indicate whether they could belong to an underdamped version of the system, an overdamped version of the system, or whether they cannot be an output signal of the system ("impossible"). Assume that the values of all system parameters are positive.

Use the table below to indicate your choices and briefly indicate the reason for your choice in each case:

a) underdamped ____, overdamped ____, impossible ____, because: _____

b) underdamped ____, overdamped ____, impossible ____, because: _____

c) underdamped ____, overdamped ____, impossible ____, because: _____

d) underdamped ____, overdamped ____, impossible ____, because: _____

e) underdamped ____, overdamped ____, impossible ____, because: _____

f) underdamped ____, overdamped ____, impossible ____, because: _____

g) underdamped ____, overdamped ____, impossible ____, because: _____

h) underdamped ____, overdamped ____, impossible ____, because: _____

i) underdamped ____, overdamped ____, impossible ____, because: _____

j) underdamped ____, overdamped ____, impossible ____, because: _____

Problems

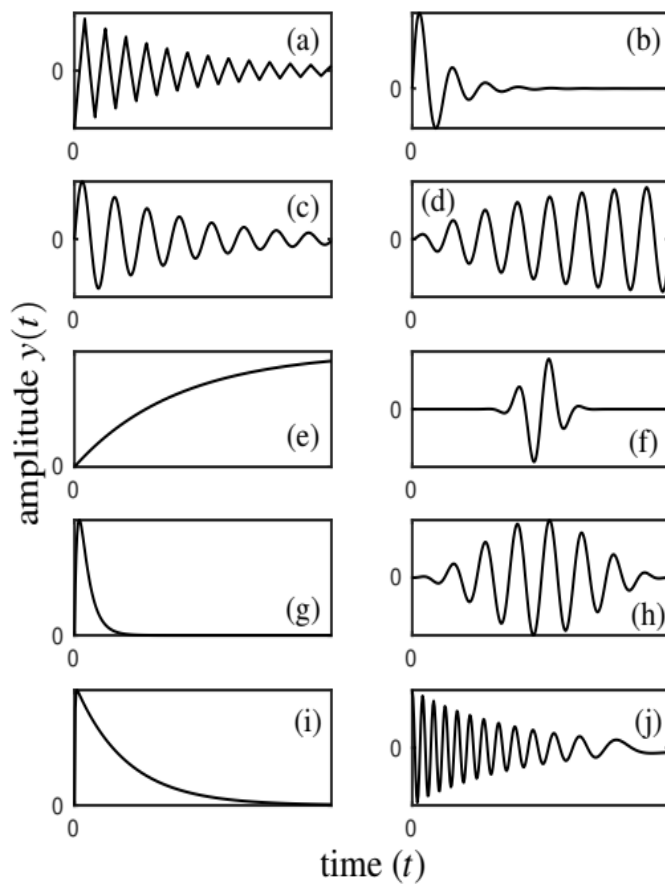


Figure 3: Candidate impulse responses (a-j) for the mechanical system (Figure 1).

Problems

4. **(30 points)** The signal in the axes at the top of Figure 4 is the sinusoidal input $u(t)$ to the mechanical system shown in Figure 1. The axes labeled (a) through (d) below the input signal are candidate steady-state responses $y(t)$ of the system. For each of the candidate output signals (a to d), indicate whether they could be a possible output or an impossible output. Briefly justify each of your answers.

a) possible output ____, impossible ____, because: _____

b) possible output ____, impossible ____, because: _____

c) possible output ____, impossible ____, because: _____

d) possible output ____, impossible ____, because: _____

e) possible output ____, impossible ____, because: _____

Problems

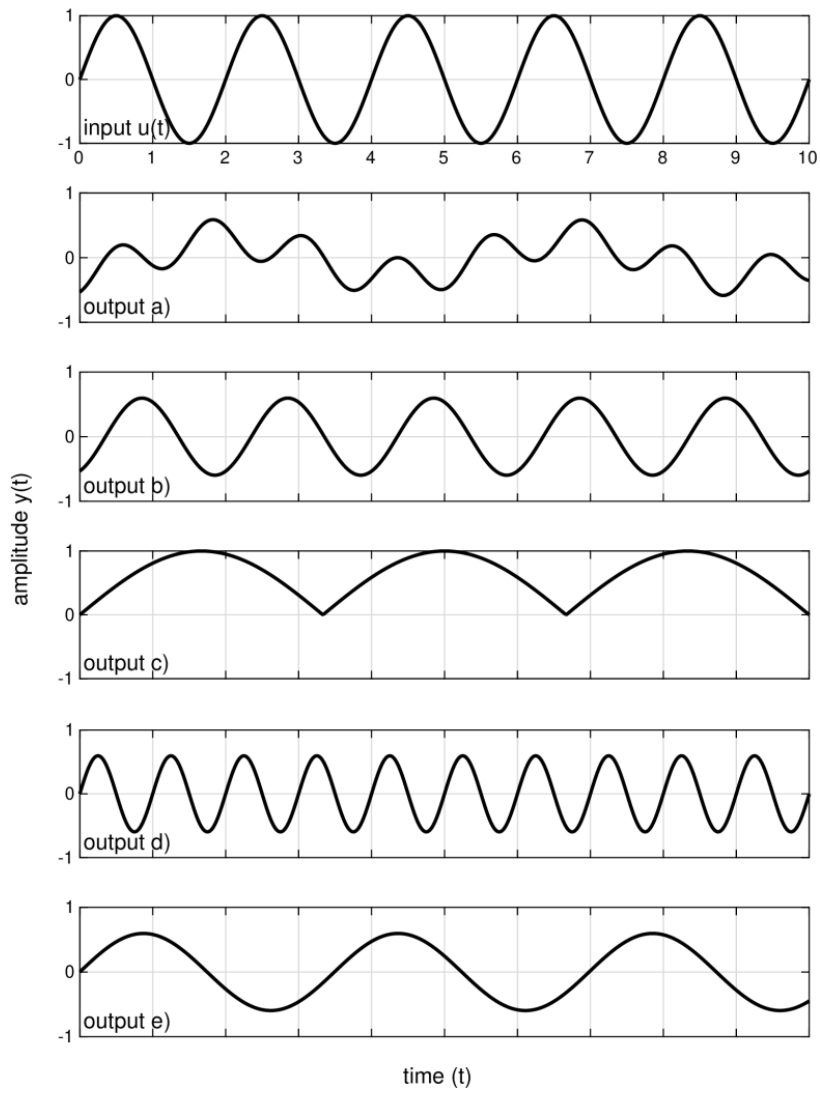


Figure 4: Candidate steady-state output signals (a-e) of the mechanical system (Figure 1) in response to a sinusoidal input (top).