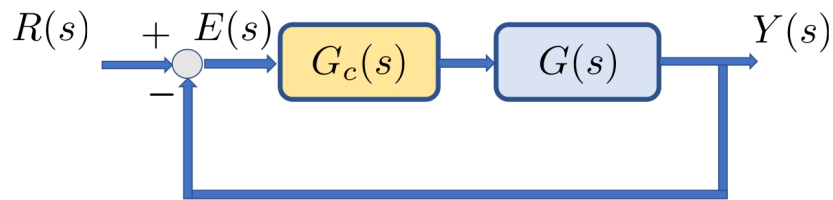

Controls - 2

Your task is to design a compensator, $G_c(s)$, for the unity closed-loop system shown below and then analyze the system properties. The controller design approach is based on the “**root locus method**”. The feedforward transfer function is given by

$$G(s) = \frac{10}{s(s + 8)} .$$



System Block Diagram

Part 1) (40 points) Consider a compensator as follows:

$$G_c(s) = \frac{K}{s + p} .$$

Using the *angle condition* in root locus approach, design the pole of the compensator such that the damping ratio ζ and the underdamped natural frequency ω_n of the dominant closed-loop poles become 0.5 and 4 (rad/sec), respectively.

Hint: You do NOT need to sketch the root locus. Please do NOT use the frequency-domain approach to design the compensator. In addition, we know that $\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$ and $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.

Part 2) (20 points) Using the *magnitude condition* in the root locus approach, obtain the gain value of the compensator (i.e., K) in Part 1.

Part 3) (20 points) Compute the steady-state tracking error (i.e., $\lim_{t \rightarrow \infty} e(t)$) for the closed-loop system that was designed in Parts 1 and 2 when we apply following input

$$r(t) = 3t - 1.$$

Problems

Part 4) (20 points) Which of the following plots (a, b, c, and d) can correspond to the root-locus sketch of the *compensated system* $G_c(s)G(s)$? Please support your answer.

Hint: You do NOT need to follow all the steps for sketching the root locus. You do NOT need to plot the root locus. Please only present the reasons that justify your choice amongst the given four plots.

