Fluids - 1

In Cartesian coordinates, consider the steady, two-dimensional (in the x and y plane), incompressible velocity field of

$$\vec{V} = (u, v) = (ax + b)\vec{i} + (-ay + cx)\vec{j}$$

where a, b and c are constants, $\vec{\imath}$ and $\vec{\jmath}$ are unit vectors in x and y coordinates, respectively.

(a) (**20 points**) Determine whether mass continuity equation is satisfied or not.

(b) (**40 points**) Determine whether the given velocity field satisfies the Navier-Stokes equation. (Hint: A physically realistic steady, incompressible flow field requires a pressure field that is a smooth function of spatial coordinates, i.e., check the values of cross-differentiation $\frac{\partial^2 P}{\partial x \partial y}$ or $\frac{\partial^2 P}{\partial y \partial x}$)

(c) (**40 points**) Calculate the pressure P as a function of x and y. (Hint: There is no need to solve for the integration constant.)

Show detailed steps in order to get full credit.

Hint: In Cartesian coordinate system of (x, y, z), fluid velocity $\vec{V} = (u, v, w)$ and continuity equation for incompressible fluid is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Conservation of momentum equations for incompressible flow in Cartesian coordinate system with gravity $\vec{g} = (g_x, g_y, g_z)$, p is pressure, ρ is fluid density and μ is fluid viscosity:

$$-\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$-\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$-\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$