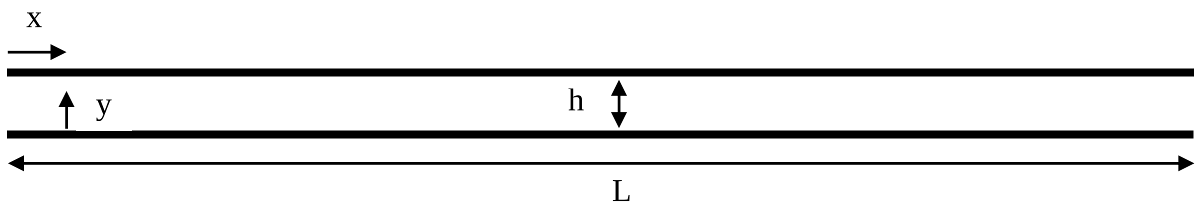
Fluids - 2

This problem will exploit dimensional analysis to study the flow of a viscous fluid between two parallel plates. Consider the geometry depicted below.



Between the two plates, a two-dimensional incompressible flow occurs in the x-y plane. Let u and v refer to the velocity component in the x and y direction, respectively. The density, pressure, and kinematic viscosity are ρ, p, and ν respectively.

1. [**30 points**] Define the characteristic flow velocities in the "x" and "y" directions as U and V, respectively. Through extensive experimentation it has been discovered that V is independent of both the pressure and the fluid properties. Using dimensional analysis, define a set of Π groups that describe V.

2. [**40 points**] Under the assumptions of Problem 1, the governing equation for the fluid velocity is,

$$\frac{\partial^2 u}{\partial y^2} = \Psi$$
$$\frac{\partial^2 v}{\partial y^2} = 0$$

   where Ψ is a constant related to the pressure gradient. Solve for the velocities subject to the boundary conditions:

$$u(x,0) = 0$$
$$u(x,h) = U$$
$$v(x,0) = 0$$
$$v(x,h) = 0$$

3. [**30 points**] Consider the case, $U = 0$, and denote the areal flow (volumetric flow rate per unit depth) between the plates as Q. Find an expression for Q in terms of Ψ and the geometric variables h, and L.