Fluids – 1

Consider a zero pressure-gradient boundary layer growing along a flat plate located in a uniform stream. The sketch shows the plate, a qualitative depiction of the stream-wise velocity profile and a representation of the boundary layer edge (the dotted line). The origin of the stream-wise coordinate, x, is the leading edge of the plate and the coordinate normal to the plate is y. The flow is two-dimensional with no variation in z, perpendicular to the page. The flow is steady and incompressible. Note that the stream-wise x component of the velocity in the free stream is U and constant along the flow.

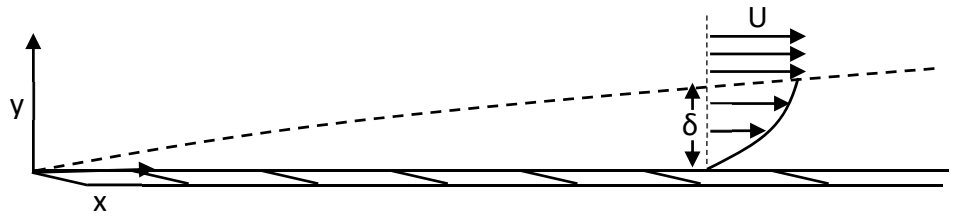

Figure 1.

As an approximation assume the streamwise velocity component, u(x,y), within the boundary layer is given by:

$$\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$$

where δ is the thickness of the boundary layer at any location x downstream of the leading edge.

a. (**18 pts**) Derive an expression for the volumetric flow rate within the boundary in terms of U and δ (per unit width in z).
b. (**12 pts**) Derive an expression for the local wall shear stress along the plate in terms of U, δ, and the viscosity, μ.
c. (**5 pts**) Consider figure 2 which shows a rectangular control volume of height H surrounding the same boundary layer as shown in figure 1. Is there any flow through the top surface of this control volume? If the answer is yes, determine an expression for the mass rate of flow out the top in terms of ρ, U and H. If the answer is no, explain.

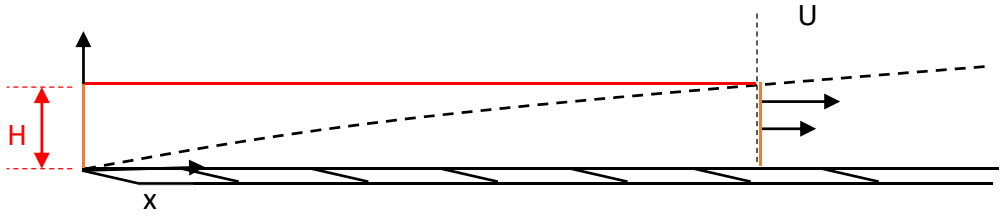

Figure 2.

d. (**40 pts**) For the control volume of figure 2 and using control volume analysis (not the local expression from part (b)), determine an expression for the "x" component of the

force exerted by the plate on fluid in the control volume in terms of U, H, and the fluid density ρ (per unit width in z).

Now dye is slowly injected into the fluid out in the free stream at the leading edge (figure 3). The dye is injected at a distance L from the plate surface. This dye follows along a streamline and is observed to gradually be displaced away from the wall as the boundary layer develops.

See figure 3.

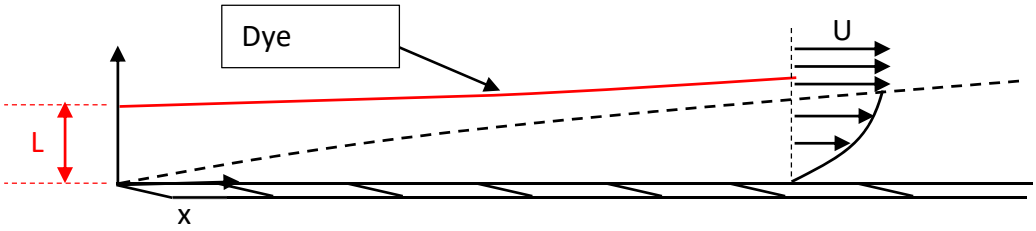

Figure 3.

e. (**15 pts**) Find an expression for the distance the streamline is displaced away from the wall in terms of δ.
f. (**10 pts**) Will the dye enter the boundary at some downstream location? If the answer is no, explain. If the answer is yes, determine an expression for δ in terms of L for the condition when the dye just enters the layer.