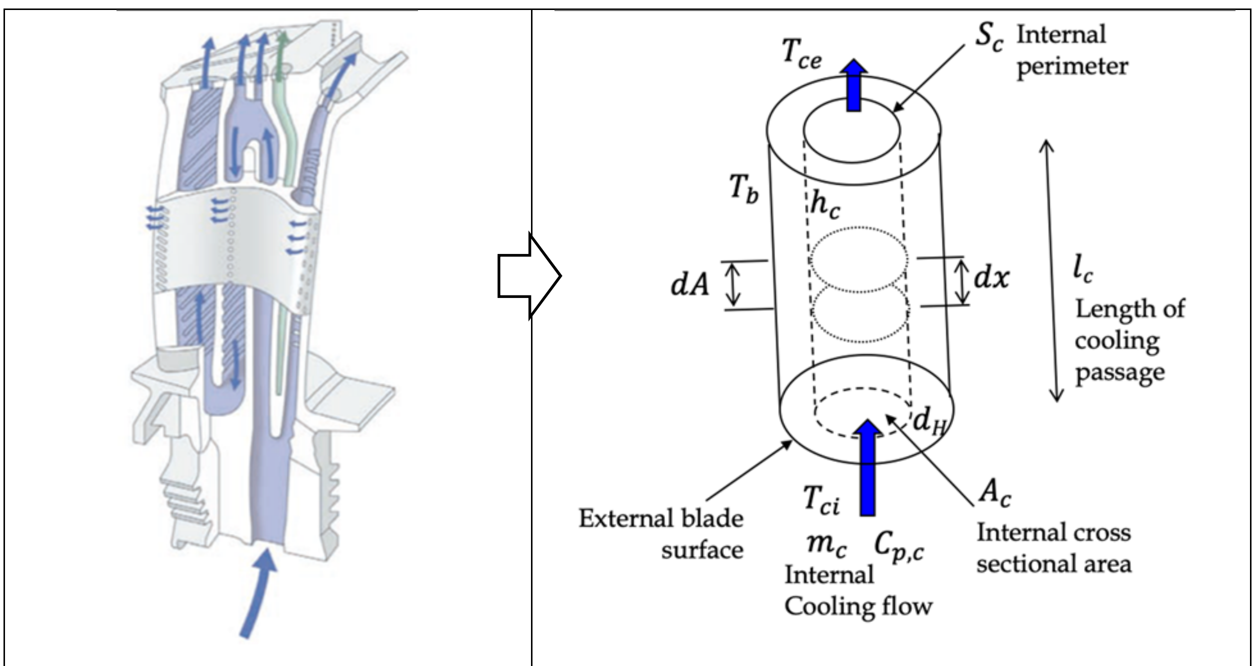
Heat Transfer - 1

Figure 1 (a) shows a typical turbine blade for aerospace application. The blade is designed with internal cooling passages as well as external film cooling holes so that the temperature of the blade can be maintained at the acceptable limit of its material.

The complicated blade geometry can be simplified as a cylindrical tube as shown in Figure 1 (b). The simplified model assumed that the blade temperature (T_b) is constant over its thickness and the internal cooling passages are represented by a cylindrical tube with cooling flow (m_c, and $C_{p,c}$). The coolant temperatures at the inlet and exit of the cylindrical tube are T_{ci} and T_{ce}, respectively.

By applying energy balance approach to the simplified model, we can drive the relationship among the initial design parameters as shown in Figure 1 (b). The most useful form is the relationship among cooling efficiency (η_c), cooling side heat transfer (Stanton number, St_c) and geometrical parameter of cooling passage (ratio of length and diameter, l_c/d_H), as shown in below.

$$\eta_c = f(St_c, l_c/d_H)$$



(a) Physical Representation (b) Simplified Model

Figure 1 Physical Representation and Simplified Model

(a) Derive the equation for the rate of bulk mean temperature increase per distance for the cooling air in the cylindrical tube $\left(\frac{dT_c}{dx}\right)$. **[30 points]**

(b) Solve this equation for the bulk mean air temperature as a function of distance, x. **[50 points]**

(c) Substitute the following expressions to the solution (b) to express this in non-dimensional form as a function of the cooling efficiency, Stanton number and the passage length/diameter ratio. **[10 points]**

- Cooling efficiency, $\eta_c = (T_{ce} - T_{ci})/(T_b - T_{ci})$
- cooling side Stanton Number, $St_c = h_c A_c / m_c C_{p,c}$
- The ratio $S_c l_c / A_c$ is proportional to the passage length/diameter ratio, $S_c l_c / A_c = 4 l_c / d_H$)

(d) Sketch the temperature distribution of the cooling air in the channel. **[10 points]**