Heat Transfer - 1

A titanium coated steel plate is subjected to a high temperature flow with free stream temperature, $T_{\infty 1}$ and ambient conditions at temperature, $T_{\infty 2}$ as shown in the figure. Assume one-dimensional flow of heat in the y-direction and no contact resistance at the interface. Denote titanium by subscript 1 and steel by subscript 2.

(a) [**10 points**] Given that heat transfer occurs through convection and conduction, sketch the expected variation of temperature for $T_{\infty 1} > T_{\infty 2}$ in the y-direction starting at $T_{\infty 1}$. Plot temperature on the vertical axis and y- on the horizontal axis demarcating the different boundaries.

(b) [**10 points**] If a titanium coating of thickness t_1 is used to coat the steel plate of thickness t_2, obtain an expression in symbolic form for the effective thermal conductivity k_{eff} of the composite plate of thickness $(t_1 + t_2)$ in terms of the relevant parameters.

(c) [**10 points**] If the heat transfer coefficient on the inner side is h_1 and on the outer side is h_2, develop an expression for an effective convective heat transfer coefficient, h_{eff} which offers the same equivalent resistance to convective heat transfer.

(d) [**10 points**] Given the properties of titanium and steel in the figure below and $t_1 = 10 \ mm$ and $t_2 = 30 \ mm$, find the effective thermal conductivity.

(e) [**10 points**] Given $h_1 = 25 \frac{W}{m^2 \cdot K}$ and $h_2 = 5 \frac{W}{m^2 \cdot K}$ find the effective heat transfer coefficient.

(f) [**10 points**] If $T_{\infty 1} = 1000 \ K$ and $T_{\infty 2} = 300 \ K$ find the heat flux in W/m^2 at steady state.

(g) [**20 points**] Find the temperature at the interface of titanium and steel.

(h) [**10 points**] If the specific heat of titanium were $C_{p1} = 100 \ J/kg \cdot K$ instead of the given value of $522 \ J/kg \cdot K$ what effect would it have on the heat flux at steady state?

(i) [**10 points**] If you wanted to increase the heat flux at steady state given the same internal and ambient temperatures, what would be the most effective strategy? Explain why.

Titanium
$\rho_1 = 4500 \ kg/m^3$
$C_{p1} = 522 \ J/kg \cdot K$
$k_1 = 21.9 \ W/m \cdot K$

Steel
$\rho_2 = 7800 \ kg/m^3$
$C_{p2} = 434 \ J/kg \cdot K$
$k_2 = 60.5 \ W/m \cdot K$

Flow, $T_{\infty 1}$, h_1
Inner side
Outer side
ambient, $T_{\infty 2}$, h_2