Machine Design - 2

Mechanical parts under cyclic loadings are prone to fatigue failure, particularly for those with stress concentrators. Fig. 1 below shows the fatigue crack formation for a second stage compressor rotor drum of an aeroengine. The close-up photo in (b) shows the fatigue cracks at the rivet hole (region marked in (a)).

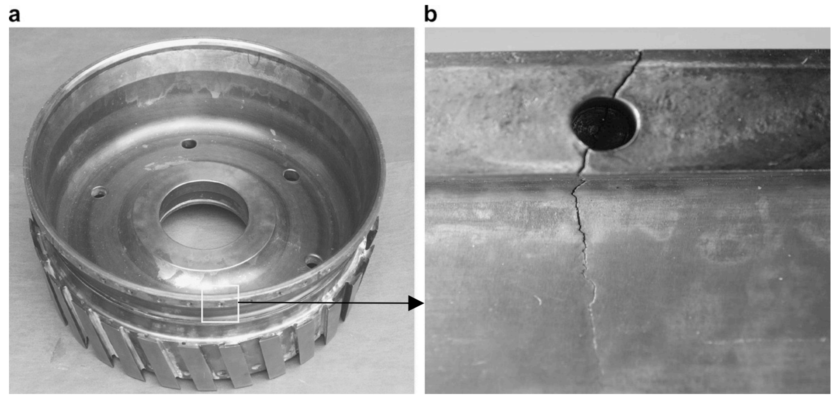


Fig. 1. Adapted from S.K. Bhaumik *et al. Engineering Failure Analysis* 15 (2008) 675–694.

In this problem, we aim to analyze the fatigue performance of mechanical parts with rivet holes by considering the following simplified model. Assume the part is a rectangular plate (width: 25 mm; thickness: 7 mm) with a through-thickness hole (diameter: 5 mm), as shown in Fig. 2. The plate is made of cold-drawn AISI 1050 steel (S_{ut} = 690 MPa; S_y = 580 MPa). The plate is subjected to an axial load fluctuating between 0 to 30 kN in tension.

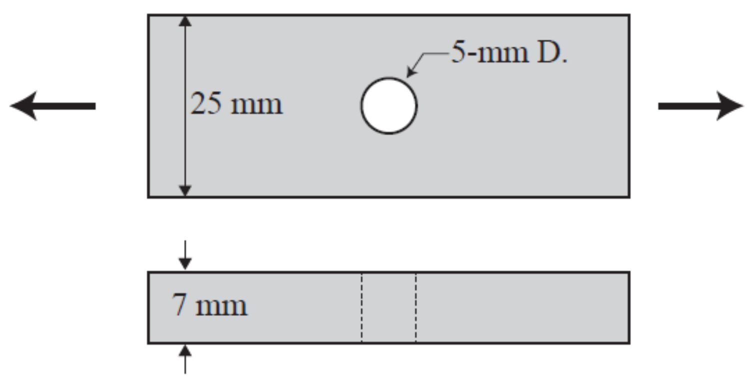


Fig. 2.

(a) (**20 points**) Estimate the endurance strength of this part if 99.99% reliability is required.
(b) (**20 points**) Estimate the fatigue stress concentration factor.
(c) (**20 points**) Determine the stress amplitude and mean stress of this part under the given loading condition.
(d) (**20 points**) Determine the factor safety against yielding failure, and fatigue failure by using the Goodman criteria.
(e) (**20 points**) If infinite life is not predicted, by using the fatigue criteria of Goodman, estimate the equivalent completely reversed stress and the predicted life.

Some useful design information is given below.

Strain-life equation	$\frac{\Delta\tilde{\varepsilon}}{2} = \frac{\sigma_f'}{E}(2N)^b + \varepsilon_f'(2N)^c$ **(6-7)**
Stress intensity range per cycle	$\Delta K_I = \beta(\sigma_{\max} - \sigma_{\min})\sqrt{\pi a} = \beta\Delta\sigma\sqrt{\pi a}$ **(6-1)**
Crack growth (Paris equation, differential form)	$\frac{da}{dN} = C(\Delta K_I)^m$ **(6-2)**
Crack growth (Paris equation, integral form)	$\int_0^{N_f} dN = N_f = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\beta\Delta\sigma\sqrt{\pi a})^m}$ **(6-3)**
Endurance limit	$S_e' = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$ **(6-10)**
High-cycle S-N line	$S_f = aN^b$ **(6-12)** $a = \frac{(fS_{ut})^2}{S_e}$ **(6-13)** $b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$ **(6-14)** $f = 1.06 - 2.8(10^{-3})S_{ut} + 6.9(10^{-6})S_{ut}^2 \quad 70 < S_{ut} < 200 \text{ kpsi}$ $f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \quad 500 < S_{ut} < 1400 \text{ MPa}$ **(6-11)** $N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b}$ **(6-15)**
Endurance limit for structural elements	$S_e = k_a k_b k_c k_d k_e S_e'$ **(6-17)**
Surface factor	$k_a = aS_{ut}^b$ **(6-18)**
Size factor (rotating bending or torsion)	$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$ **(6-19)**

Size factor (Axial loading)	$k_b = 1$
Load factor	$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$ **(6-25)**
Temperature factor	$k_d = S_T / S_{RT}$ **(6-27)** $S_T / S_{RT} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$ $S_T / S_{RT} = 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2$ **(6-26)**
Fatigue stress concentration	$\sigma_{max} = K_f \sigma_0$ or $\tau_{max} = K_{fs} \tau_0$ **(6-30)**
Fatigue stress concentration factor	$K_f = 1 + q(K_t - 1)$ or $K_{fs} = 1 + q_s(K_{ts} - 1)$ **(6-32)**
Notch sensitivity	$q = \dfrac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}}$ **(6-33)**
$\sqrt{a}$ **(bending or axial)**	$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \le S_{ut} \le 250$ kpsi $\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \le S_{ut} \le 1700$ MPa **(6-35)**
$\sqrt{a}$ **(torsion)**	$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \quad 50 \le S_{ut} \le 220$ kpsi $\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \quad 340 \le S_{ut} \le 1500$ MPa **(6-36)**
Mean stress	$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$ **(6-9)**
Stress amplitude	$\sigma_a = \left\lvert \dfrac{\sigma_{max} - \sigma_{min}}{2} \right\rvert$ **(6-8)**
Langer yield criterion	$n_y = \dfrac{S_y}{\sigma_{max}} = \dfrac{S_y}{\sigma_a + \lvert\sigma_m\rvert}$ **(6-43)**
Goodman criterion	Design equation: $n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}\right)^{-1} \quad \sigma_m \ge 0$ **(6-41)**
Morrow criterion	Design equation: $n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{\tilde{\sigma}_f}\right)^{-1}$ or $n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{\sigma_f'}\right)^{-1}$ **(6-46)**

Gerber criterion	Design equation: $n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \quad \sigma_m \ge 0$ **(6 - 48)**
Soderberg criterion	Design equation: $n = \left(\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}\right)^{-1} \quad \sigma_m \ge 0$ **(6 - 50)**
ASME-Elliptic criterion	Design equation: $n_f = \left[\left(\frac{\sigma_a}{S_e}\right)^2+\left(\frac{\sigma_m}{S_y}\right)^2\right]^{-1/2} \quad \sigma_m \ge 0$ **(6 - 52)**
Smith-Watson-Topper (SWT) criterion	Design equation: $n_f = \frac{S_e}{\sqrt{(\sigma_m+\sigma_a)\sigma_a}}$ **(6 - 54)**
Walker criterion	Design equation: $n_f = \frac{S_e}{(\sigma_m+\sigma_a)^{1-\gamma}\sigma_a^{\gamma}}$ **(6 - 56)** $\gamma = -0.0002S_{ut}+0.8818 \quad (S_{ut}\text{ in MPa})$ $\gamma = -0.0014S_{ut}+0.8818 \quad (S_{ut}\text{ in kpsi})$ **(6 - 57)**
Equivalent completely reversed stress (Goodman)	$\sigma_{ar} = \frac{\sigma_a}{1-\sigma_m/S_{ut}}$ **(6 - 59)**
Combined loading	$\sigma'_m = \left\{\left[(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}}+(K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}\right]^2+3\left[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}\right]^2\right\}^{1/2}$ **(6 - 67)** $\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}}+(K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}\right]^2+3\left[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}\right]^2\right\}^{1/2}$ **(6 - 66)**
Accumulated damage	$D = \sum \frac{n_i}{N_i}$ **(6 - 69)**

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C. J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis,* vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook,* McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

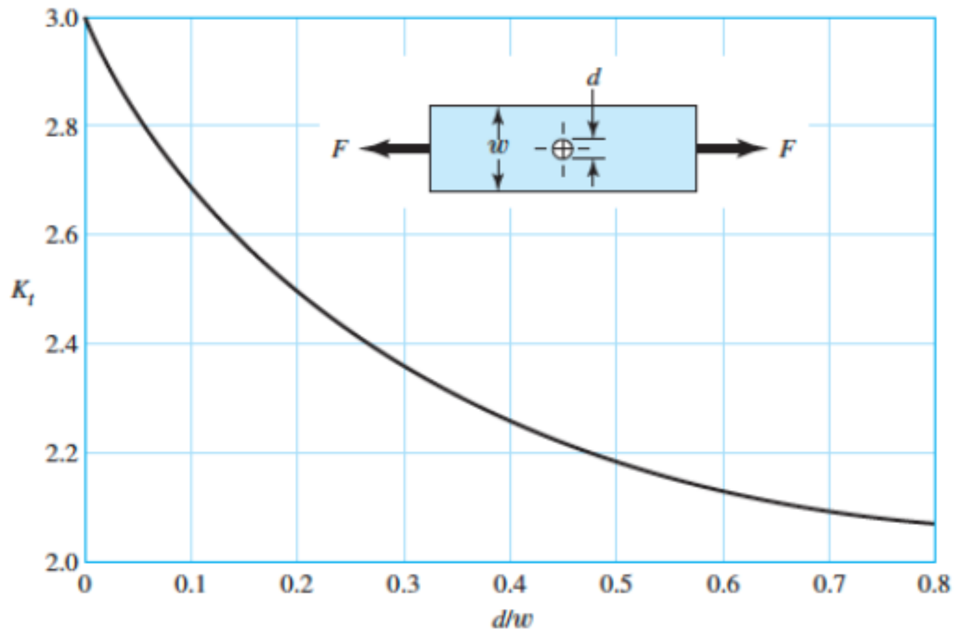
Table 6–5

Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Figure A–15–1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



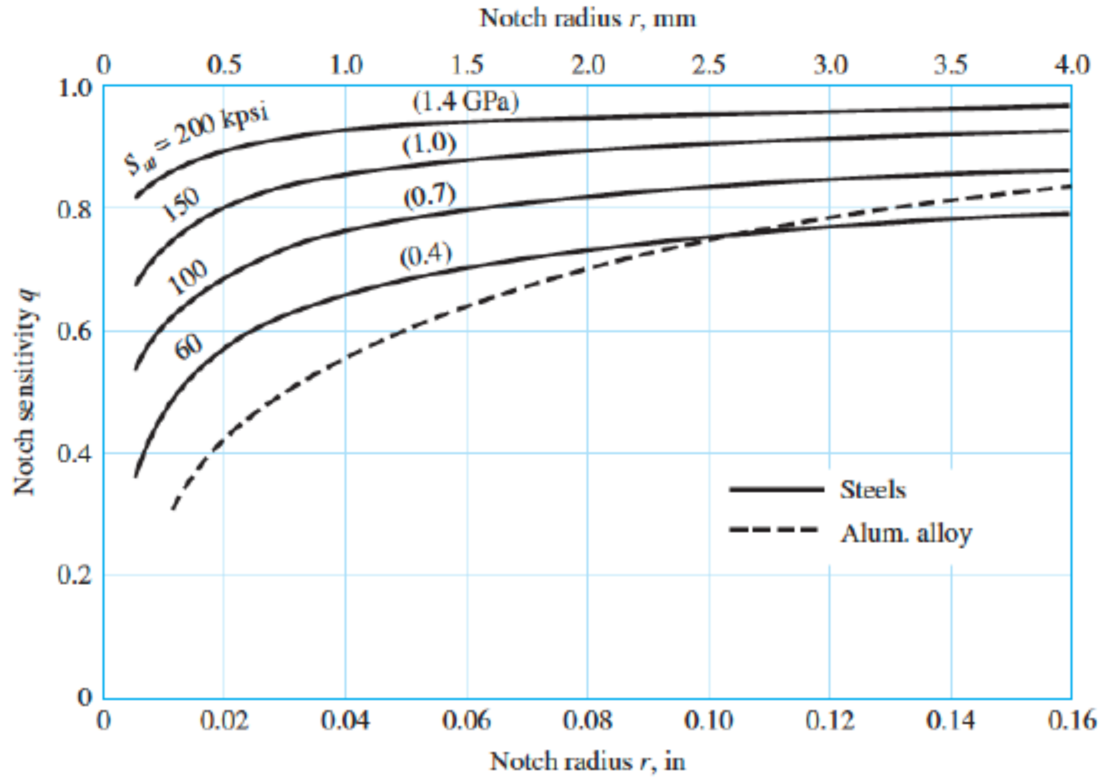


Figure 6–20

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S'_e = 0.5S_{ut}$ at 10^6 cycles.