Mathematics - 1

Part a (50 points)

1. **(10 points)** Let $g(x) = ax^2 + bx + c$ what is $\frac{dg(x)}{dx}$?

2. **(20 points)** Let $F(y) = \frac{1}{\sqrt{y}}$ what is $\frac{dF(y)}{dy}$?

3. **(20 points)** Substituting $g(x)$ into y, what is $\frac{dF(x)}{dx}$?

Part b (50 points)

For two point bodies *i* and *j*, let $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ be position coordinate vectors in 3D:

$$\boldsymbol{r}_i = \begin{bmatrix} r_{ix} \\ r_{iy} \\ r_{iz} \end{bmatrix} \qquad \boldsymbol{r}_j = \begin{bmatrix} r_{jx} \\ r_{jy} \\ r_{jz} \end{bmatrix} \tag{2}$$

The Euclidean norm between the bodies is given by:

$$\|\boldsymbol{r}_i - \boldsymbol{r}_j\| = \sqrt{(r_{ix} - r_{jx})^2 + (r_{iy} - r_{jy})^2 + (r_{iz} - r_{jz})^2} \tag{3}$$

1. **(20 points)** Expand the Euclidean norm into the form $\sqrt{ax^2 + bx + c}$ and substitute r_{ix} for x. What are *a*; *b*; *c* (5 points each) in terms of the remaining elements of $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$?

2. **(30 points)** Let $F(\boldsymbol{r}_i, \boldsymbol{r}_j) = \frac{1}{\|\boldsymbol{r}_i - \boldsymbol{r}_j\|}$. Show that:

$$\frac{dF(\boldsymbol{r}_i, \boldsymbol{r}_j)}{dr_{ix}} = -\frac{(r_{ix} - r_{jx})}{\|\boldsymbol{r}_i - \boldsymbol{r}_j\|^3} \tag{4}$$