
Mathematics - 2

1. The transformation relating the cartesian coordinates x, y, z to the parabolic coordinates u, v, θ is given by $x = u v \cos \theta$, $y = u v \sin \theta$, and $z = (u^2 - v^2)/2$, where $(u \geq 0, v \geq 0)$.
(30 points) Verify that the parabolic coordinate system is orthogonal
2. **(40 points)** Write the following 2nd order differential equation as a system of 1st order, linear differential equations.

$$2y'' - 5y' + y = 0 \quad y(3) = 6 \quad y'(3) = -1$$

3. Let $\mathbf{F} = \sin x \mathbf{i} + \cos y \mathbf{j} + \ln z \mathbf{k}$ ($z \neq 0$), and $\mathbf{G} = e^x \cos y \mathbf{i} + e^x \sin y \mathbf{j}$.
- Find
- (a) **(10 points)** $\mathbf{F} + \mathbf{G}$
 - (b) **(10 points)** $\mathbf{F} \cdot \mathbf{G}$
 - (c) **(10 points)** $\mathbf{F} \times \mathbf{G}$