Math - 2

Problem 1 [**25 Points**]
Given

$$u(x) = \frac{1}{a^2}[1 - e^{-ax} - x(1 - e^{-a})]$$

determine $u(x)$ for $a = 0$.

Problem 2
Evaluate the following integrals analytically

(a) [**20 Points**] $\int_0^1 \cos \lambda_m x \cos \lambda_n x \, dx \,; \quad \lambda_m = \left(m + \frac{1}{2}\right)\pi, \lambda_n = \left(n + \frac{1}{2}\right)\pi$

m and *n* are integers. Consider any general *n* and *m*, including *n* = *m*

(b) [**15 Points**] $\int_0^1 x^2 \cos n\pi x \, dx$

Express your final answer without any trigonometric expression.

Problem 3 [**40 Points**]
Find all the roots of the equation $x^6 - 1 = 0$, analytically. Do not use a graphing calculator. To get credit, you need to show all steps clearly.

Hints are available from the proctor for a penalty of 5 points per hint provided.