Math - 1

Consider the linear first order ODE where τ is constant and $S(t)$ is a time varying source.

$$\frac{d\theta}{dt} = -\frac{1}{\tau}\theta + S(t)$$
$$\theta = \theta_0 \quad , \quad t = 0$$

a) Determine the general analytical solution. **(30 points)**

b) Determine the analytical solution when $\theta_0 = S = 0$. **(5 points)**

c) Determine the analytical solution when $S = 0$. **(5 points)**

d) Determine the analytical solution $S(t) = S_c$, where S_c is a constant, with a zero initial condition, $\theta_0 = 0$. **(20 points)**

e) Determine the analytical solution for the case where $\theta_0 = 0$ with a finite pulse source,

$$S(t) = \frac{S_c}{\Delta t} H(t - \Delta t) = \begin{cases} \frac{S_c}{\Delta t} & , \quad 0 \le t \le \Delta t \\ 0 & , \quad t > \Delta t \end{cases}$$

where S_c is a constant and H(t) is the unit step function. Create a well-labeled sketch of this solution. **(20 points)**

f) Examine the limit as $\Delta t \to 0$ of the solution derived in part (d). Sketch this solution. **(10 points)**

g) Express the solution for the pulsed problem in part (e) in terms of the constant source problem in part (d). **(10 points)**