PhD Qualifying Exam
Nuclear Engineering Program

Part 1 – Core Courses
(Solve 3 problems only)

9:00 am – 12:30 pm, Oct 30, 2020

(1) Nuclear Reactor Analysis

Consider a 1-D homogeneous reactor placed in a vacuum. To determine the eigenvalue and multigroup flux distributions in this reactor, you can use the multigroup diffusion equations given by

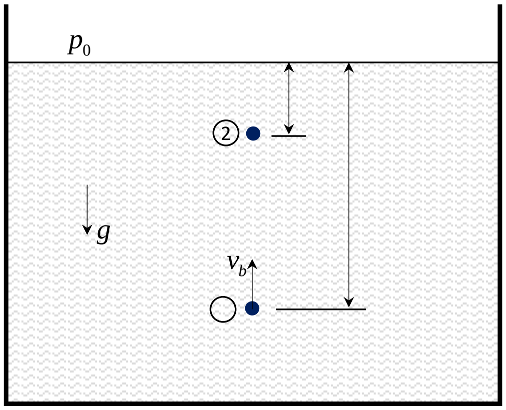
$$-D_g \nabla^2 \phi_g(\vec{r}) + \Sigma_{R,g}\phi_g(\vec{r}) = \frac{\chi_g}{k}\sum_{g'=1}^{G} \nu\Sigma_{fg'}\phi_{g'}(\vec{r}) + \sum_{\substack{g'=1 \\ g'\neq g}}^{G} \Sigma_{sg'\rightarrow g}\phi_{g'}(\vec{r}), \qquad g = 1, G$$

a) (10%) How do you determine a multigroup cross section, e.g., $\Sigma_{R,g}$?
b) (10%) What is the physical definition of group flux, $\phi_g(\vec{r})$?
c) (20%) Derive 1-D, 3-group diffusion equations, considering the following conditions:
   - Fission neutrons are generated only in the 1st energy group,
   - fission reaction happens only in the 3rd energy group, and
   - upscattering does NOT occur.
   - Directly coupled downscattering.

d) (50%) Assuming the same value of extrapolation distance for all energy groups, derive the criticality condition for this reactor using the 3-group diffusion equations you derive in part *c*.
e) (10%) Rewrite the equation you obtain in part d in terms of a set of parameters similar to those introduced in the 6-factor formula.

(2) Reactor Thermal Hydraulics

Consider a spherical air bubble rising vertically in a large water pool. At a depth h_1 from the free surface, the bubble has a diameter d_1. The bubble can be modeled as an ideal gas, and the temperature of the bubble is maintained at a constant T throughout the process. The following quantities are known: water density ρ_f, water viscosity μ_f, atmospheric pressure p_0, gravitational acceleration g, the molar mass of air M, and the universal gas constant is $\bar{R}$.



a. (10%) Draw the force diagram of the bubble, including possible unsteady forces.

b. (40%) If bubble acceleration and unsteady forces are negligible, develop an expression of the bubble velocity v_b at point 1 using the given quantities. The drag coefficient of the bubble is given by:

$$C_D = \frac{24}{N_{Re}}$$, where N_{Re} is the particle Reynolds number.

c. (15%) What is the bubble diameter when it arrives at point 2, which is at a depth h_2 from the free surface? You can neglect the surface tension force and partial pressure of water vapor, and assume the bubble's pressure is the same as the hydrostatic head.

d. (35%) What is the bubble velocity at point 2? Is it greater or smaller compared with that at point 1?

(3) Advanced Nuclear Materials

1. (50%) Describe the formation of He bubbles, voids, and cavities in materials.

2. (50%) Describe the effect of bubbles, voids, and cavities in mechanical properties.

(4) Radiation Detection and Shielding

You are tasked with determining the intrinsic peak efficiency, ε_{ip}, of a high purity germanium detector (HPGE) at a high gamma-ray energy. You determine that a calibrated reference source of Europium-152, ^{152}Eu, can be used. It emits a 1408.013 keV gamma-ray with a branching ratio of 20.87%. In the National Nuclear Data Center Chart of Nuclides, it indicates that the half-life of Eu-152 is 13.517 $\pm$ 0.014 years (one standard deviation). The calibrated reference source lists the activity of your Eu-152 source as 0.984 μCi $\pm$ 3.1% (fractional standard deviation) dated on 5/12/2014. You are counting the reference source today on 10/30/2020. This means the reference source is 6.458 years old as of today. You will need to determine the actual activity of the reference source for today.

The Eu-152 source is placed on a stand 25 cm directly above the center of the germanium detector face. The detector face has a diameter of 8.9 cm. The solid angle Ω (in steradians) subtended by the detector at the source can be determined from the following formula:

$$\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right),$$

where d is the distance that the source is from the detector face and a is the radius of the detector face.

a) (60%) You count the reference source for 5 minutes in a shielded very low background (about zero) and obtain 10,322 counts under the 1408.013 keV photopeak. Calculate the intrinsic peak efficiency of this HPGE detector at the 1408.013 keV energy.

b) (40%) Using the given uncertainty in the calibrated source activity, the uncertainty in the half-life value of the Eu-152, and the uncertainty in your measured counts, determine the uncertainty in your calculated intrinsic peak efficiency from part (a) above.

(5) Advanced Engineering Mathematics

Define $\mu = cos\,\theta$ where θ is the neutron scattering angle from a nucleus with atomic mass number A following elastic scattering. Then, $\bar{\mu}$ is the average cosine of the scattering angle.

The neutron scattering angle can be represented in different reference frames such as the lab frame or the center of mass frame, i.e. θ_L or θ_c. A relationship between the two angles can be derived giving the following expression:

$$cos\,\theta_L = \frac{1 + A\,cos\,\theta_C}{\sqrt{A^2 + 2A\,cos\,\theta_C + 1}} \qquad (1)$$

One can also derive an equation for $\bar{\mu}$ in terms of the neutron scattering angle in both the lab and center of mass frames resulting in:

$$\bar{\mu} = \frac{1}{2}\int_0^{\pi} sin\,\theta_C\,cos\,\theta_L\,d\theta_C \qquad (2)$$

We note that $\bar{\mu}$ shows up in the approximation for the diffusion coefficient:

$$D = \frac{1}{3\Sigma_S(1 - \bar{\mu})}$$

(100%) Solve the integral in equation (2) by substituting equation (1) into the integral to find an expression for $\bar{\mu}$ as a function of A only, i.e. the atomic mass number of the nucleus that the neutron scatters from.

Some Trigonometric Identities

$sin\,2\theta = 2\,sin\,\theta\,cos\,\theta$

$sin^2\,\theta + cos^2\,\theta = 1$

$sin^2\frac{\theta}{2} = \frac{1 - cos\,\theta}{2}$

$cos^2\frac{\theta}{2} = \frac{1 + cos\,\theta}{2}$