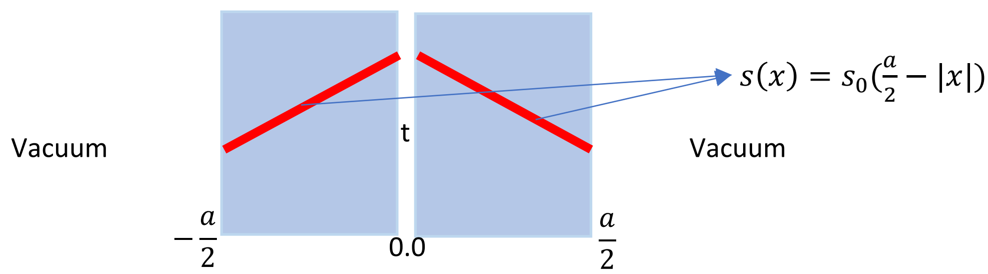
**PhD Qualifying Exam**
**Nuclear Engineering Program**

Part 1 – Core Courses
(Solve 3 problems only)

9:30 am – 1:00 pm, Apr 17, 2020

**(1) Nuclear Reactor Analysis**

A slab of size ($a$) contains a discontinuous linear source distribution and a gap (vacuum) of thickness ($t$) at its center as depicted below




If the slab is placed in a vacuum, using one-speed diffusion equation, solve for flux distribution within the slab.

**Hint:** Consider the one-speed diffusion equation is expressed by: $-D\nabla^2\phi(x) + \Sigma_a\phi(x) = S(x)$, and the following trigonometric identities:

$$\sinh(x \pm y) = sinhx \cdot coshy \pm coshx \cdot sinhy$$
$$\cosh(x \pm y) = coshx \cdot coshy \pm sinhx \cdot sinhy$$

Also, note that: $sinhx = \frac{e^x - e^{-x}}{2}$ and $coshx = \frac{e^x + e^{-x}}{2}$.

**(2) Reactor Thermal Hydraulics**

For adiabatic, air-water, two-phase upflows in a vertical pipe,

a) (40%) Sketch the four major flow regimes, discuss their characteristics.

b) (40%) Briefly describe the field equations, primary unknown variables, and important closure models in the two-fluid model.

c) (20%) Consider a single bubble in an infinite medium, sketch the drag coefficient curve covering the spherical, distorted, and cap bubbles.

**(3) Advanced Nuclear Materials**

In electrochemical separation using molten salt for spent fuel reprocessing, Pu is always separated together with another element, for example, U. In a molten salt system, the following parameters are known:

The operation temperature: T
The stable ions: $U^{3+}$ and $Pu^{3+}$
The apparent potentials: $E_{U^{3+}/U^0}$ ($E_U$) and $E_{PU^{3+}/PU^0}$ ($E_{Pu}$)
The concentration in mole fraction: $X_U^b$ and $X_{Pu}^b$
The solid cathode potential: $E_c$
The cathode surface: A
The total amount of salt in Mole: M
The total volume of the salt: V
The mass transfer coefficient $U^{3+}$ and $Pu^{3+}$: $K_m$

All the units are in SI.

1. (50%) If the surface concentration of $U^{3+}$ and $Pu^{3+}$ are uniform at the cathode surface, deduce the expression of surface concentration ratio $X_U^s$ / $X_{Pu}^s$ ($X_U^s$ and $X_{Pu}^s$ are surface concentrations of U and Pu ions on the electrode surface, respectively) using the known parameters.

2. (50%) Based on the known parameters, deduce the expression of the total current due to diffusion at cathode.

**(4) Radiation Detection and Shielding**

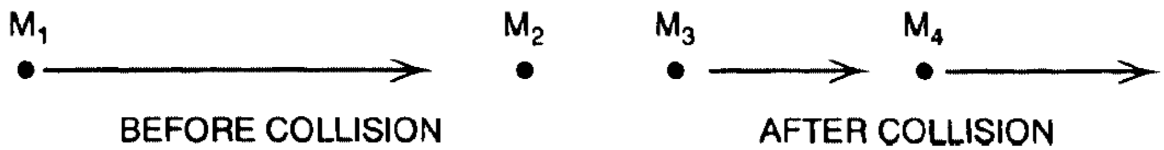



Figure 1. Schematic representation of a head-on collision producing a nuclear reaction in which the identity of the particles can change. Particle $M_1$ is the incoming particle, in this case a neutron. Particle $M_2$ is the target nucleus and is at rest before the reaction. Both particles $M_3$ and $M_4$ are in motion after the reaction where $M_3$ is the resulting product nucleus and $M_4$ is the resulting outgoing particle. The nuclear reaction is thus $M_1 + M_2 \rightarrow M_3 + M_4$.

**<u>Problem</u>**:

An endothermic reaction, by definition, requires the addition of energy in order to take place. The reaction thus converts energy into mass. Such a reaction can be brought about by one particle striking another at rest, provided the incident particle has sufficient energy. The condition for the threshold energy for a neutron reaction is different than that of a photon reaction. The neutron must have enough energy to supply both the increase in mass, i.e., the minus $Q$-value of the reaction, and also the continued motion of the center of mass of the colliding particles after the collision.

To calculate the threshold energy of the reaction we consider a head-on collision. A particle with mass $M_1$ (in this case a neutron) strikes a particle with mass $M_2$ initially at rest. The identity of the particles is changed by the reaction, and so there will generally be different masses, $M_3$ and $M_4$, after the encounter. The collision is shown schematically in Figure 1 above. The change in rest-mass energy, $Q = [M_1 + M_2 - (M_3 + M_4)]*931.5$ MeV/amu is negative for the endothermic reaction. Conservation of total energy requires that $Q = E_3 + E_4 - E_1$ where $E_i$ is the kinetic energy of the associated moving particles (where $E_2 = 0$). Conservation of momentum gives $p_1 = p_3 + p_4$, where $p^2 = 2ME$.

a. (70%) Derive an equation for the minimum threshold kinetic energy value of the neutron, $E_1$, needed in terms of $Q$, $M_1$, $M_3$, and $M_4$ for this reaction to occur using conservation of total energy and momentum.

b. (30%) The existence of $^{32}S$ in human hair has been used to help estimate high energy neutron doses to persons exposed in criticality accidents. Using your answer from part (a), calculate the threshold energy for the reaction that occurs when sulfur reacts with a fast neutron: $^{32}S(n, p)^{32}P$, where $M_n$ = 1.008665 amu, $M_{S\text{-}32}$ = 31.972071 amu, $M_{H\text{-}1}$ = 1.007825, $M_{P\text{-}32}$ = 31.973907 amu. [Note if unable to complete Part (a), at least calculate the $Q$-value for this reaction and estimate a value for the threshold energy of the neutron knowing that $E_{threshold} > |Q|$.]

**(5) Advanced Engineering Mathematics**

Xenon-135 is a strong fission product poison produced in a nuclear reactor and will impact the net reactivity in the core. It is produced directly by fission and by decay of another fission product Iodine-135. The following parent-daughter decay chain results from the Iodine:

$$^{135}\mathrm{I} \underset{6.7\ \mathrm{hr}}{\overset{\beta-}{\longrightarrow}} {}^{135}\mathrm{Xe} \underset{9.2\ \mathrm{hr}}{\overset{\beta-}{\longrightarrow}} {}^{135}\mathrm{Cs}$$

Xenon-135 is lost through radioactive decay and burnup (neutron flux absorption). When the reactor is shutdown, production through fission and burnup through absorption stops. Following reactor shutdown, the resulting equations for the atom concentrations of Iodine and Xenon are as follows:

$$\frac{dI}{dt} = -\lambda_I I, \text{ and}$$
$$\frac{dX}{dt} = \lambda_I I - \lambda_X X$$

where $I$ = the atom concentration of Iodine-135, $X$ = the atom concentration of Xenon-135 (atoms/cm$^3$).

If the reactor had been operating at 100% power for the last year prior to shutdown, then just after shutdown, $X(0) = X_\infty = 6.254 \times 10^{13}$ atoms/cm$^3$, and $I(0) = I_\infty = 2.783 \times 10^{15}$ atoms/cm$^3$, where $X_\infty$ is the equilibrium xenon concentration at time of shutdown and $I_\infty$ is the equilibrium iodine concentration at time of shutdown.

a) (60%) Derive an equation for $X(t)$ for the xenon concentration in the reactor following shutdown in terms of $X_\infty$ and $I_\infty$.

b) (40%) At what time in hours after reactor shutdown does the xenon concentration reach a maximum?

Note: If the magnitudes of the total rod worth and the total chemical shim worth from boric acid addition are less than the negative reactivity added at the time of peak xenon concentration following the shutdown, then this would result in a "xenon-precluded startup" and the reactor could not be started up for a finite duration called the "deadtime".