PhD Qualifying Exam
Nuclear Engineering Program

Part 1 – Core Courses
(Solve 3 problems only)

8:30 am – 12:00 pm, April 2, 2021

(1) Nuclear Reactor Analysis

For a one-region cylindrical reactor with size (R*H) placed in a vacuum, use the one-speed diffusion equation given by

$$-D\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi(r,z)}{\partial r}\right)+\frac{\partial^2\phi(r,z)}{\partial z^2}\right]+\Sigma_a\phi(r,z)=\frac{1}{k}\nu\Sigma_f\phi(r,z),$$

and

a) (80%) Derive a formulation for the flux distribution if the reactor power is *P*
b) (20%) Derive a formulation for the criticality condition

Hint: Bessel's differential equations are given by

$$x^2\frac{d^2f(x)}{dx^2}+x\frac{df(x)}{dx}+(x^2-n^2)f(x)=0,$$

and the *modified* Bessel's differential equations are given by

$$x^2\frac{d^2f(x)}{dx^2}+x\frac{df(x)}{dx}-(x^2+n^2)f(x)=0.$$

The solutions to the Bessel's equations and their modified forms are given in the attachment.

Also consider the following identities:

- For derivatives of the Bessel's functions and the *modified* Bessel's functions:

$$\frac{dJ_0(x)}{dx}=-J_1(x),\qquad \frac{dY_0(x)}{dx}=-Y_1(x),\quad \frac{dI_0(x)}{dx}=I_1(x),\quad \frac{dK_0(x)}{dx}=-K_1(x)$$

- For integrals of the Bessel's functions and the *modified* Bessel's functions:

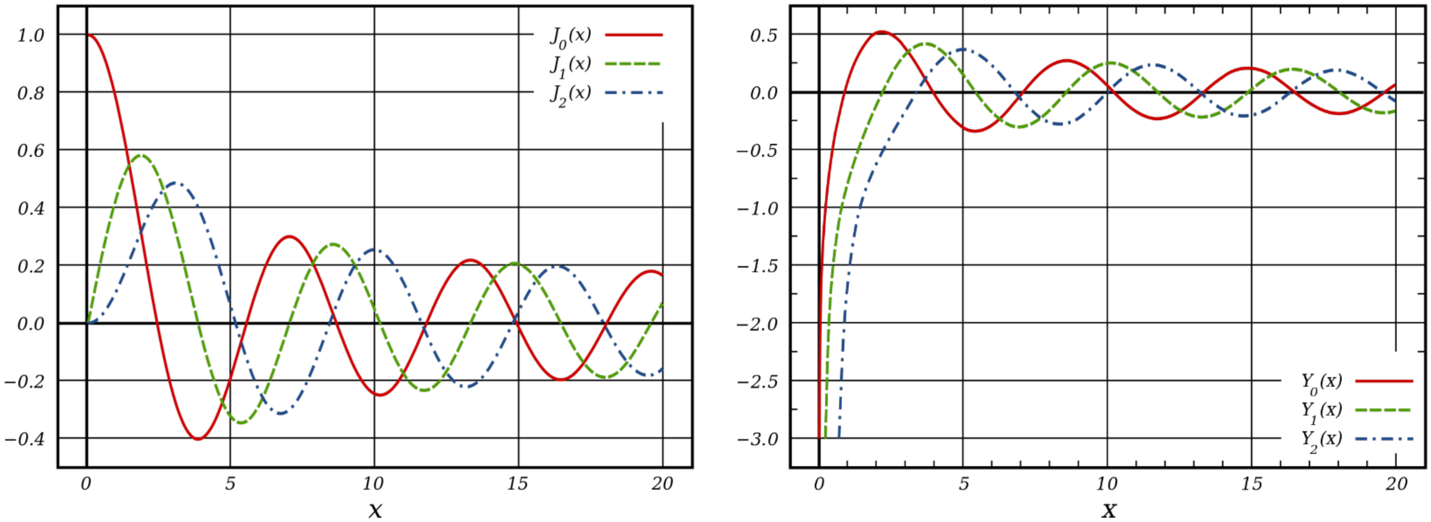
$$\int dx\, x^nJ_{n-1}(x)=x^nJ_n(x),\int dx\, x^nY_{n-1}(x)=x^nY_n(x)$$

$$\int dx\, x^nI_{n-1}(x)=x^nI_n(x),\int dx\, x^nK_{n-1}(x)=x^nK_n(x)$$

Attachment

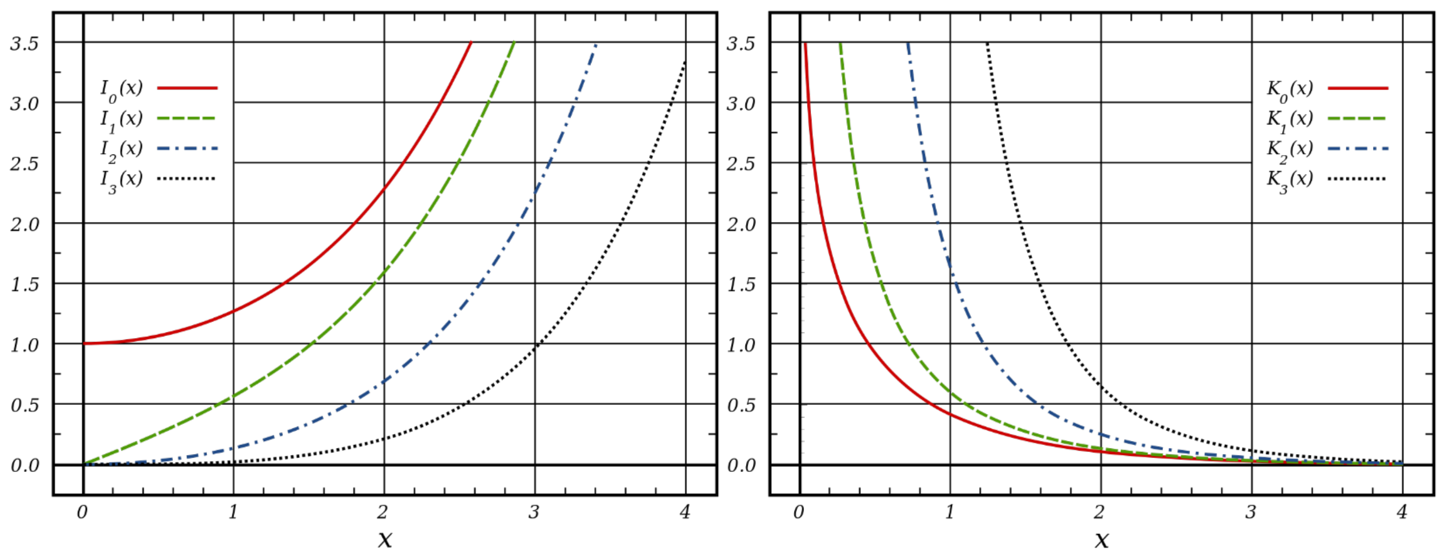
Bessel Functions

[$J_n(x)$: 1[st] kind (left) & $Y_n(x)$: 2[nd] kind (right)]



Modified Bessel Functions

[$I_n(x)$: 1[st] kind (left) & $K_n(x)$: 2[nd] kind (right)]



(2) Reactor Thermal Hydraulics

A spherical fuel pebble in a gas-cooled reactor has been operated at a power of p_0 and a temperature of T_0 for an essentially infinite period before shutdown. Initially, no cooling is available to remove the decay heat. From $t = t_1$, helium gas with a temperature of T_∞ is used to cool the pebble. The heat transfer coefficient h is maintained at a constant level. The pebble diameter d, density ρ, and specific heat c_v are all known constants.

a) (30%) Determine the pebble temperature at time $t = t_1$.
b) (30%) For $t > t_1$, write the pebble temperature equation based on energy balance. You do not need to solve the equation.
c) (40%) Determine the pebble temperature when a sufficiently long time t_2 has passed after shutdown.

Note: All the times are given in seconds after shutdown. Temperature variation within the pebble can be neglected. The decay power p_d can be approximated by the following equation:

$$p_d \approx 0.066 p_0 t^{-0.2}.$$

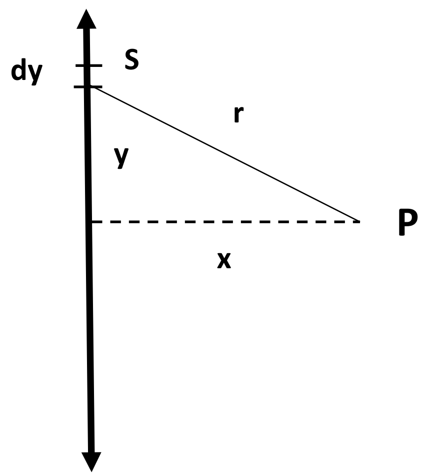
(3) Advanced Nuclear Materials

a) (50%) Why do materials experience radiation induced precipitation (RIP)?

b) (50%) What are the effects of RIP on mechanical properties?

(4) Radiation Detection and Shielding

a) (50%) A radioactive line source is shown in the figure below.



Assume that the differential line element *dy* is a point source **S,** where **S** represents the number of gamma-rays per second per unit distance along the line. Then we can write the differential gamma-ray intensity at point **P** due to *dy* as

$dI(r) = \frac{Sdy}{4\pi r^2}$, where $r^2 = x^2 + y^2$.

We want to show that the gamma-ray intensity falls off as a function of 1/x with distance from a line source. This is different than for a point source where the intensity falls off as $1/x^2$.

Derive an equation for $I(x)$ for an infinite line source where

$I(x) = \int_{-\infty}^{+\infty} \frac{Sdy}{4\pi r^2}$.

You may use the following Integral Table as needed.

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2 + x^2|$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$$

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b) (50%) Assume for a fast reactor that we have a long 20 cm diameter pipe containing liquid sodium coolant leaving the reactor. The sodium is activated by the fast neutron flux as it passes through the core producing the ^{24}Na radioisotope. ^{24}Na undergoes β^- decay to ^{24}Mg with a 15 hour half-life and also emits two gamma-rays of 1.37 MeV and 2.75 MeV with every decay. The specific activity concentration of ^{24}Na in the coolant is 0.5 Ci/cm^3.

For calculational purposes it is convenient to express the buildup factor used in shielding as a mathematical function. One of the most useful of these is a sum of exponentials known as the Taylor form of the buildup factor:

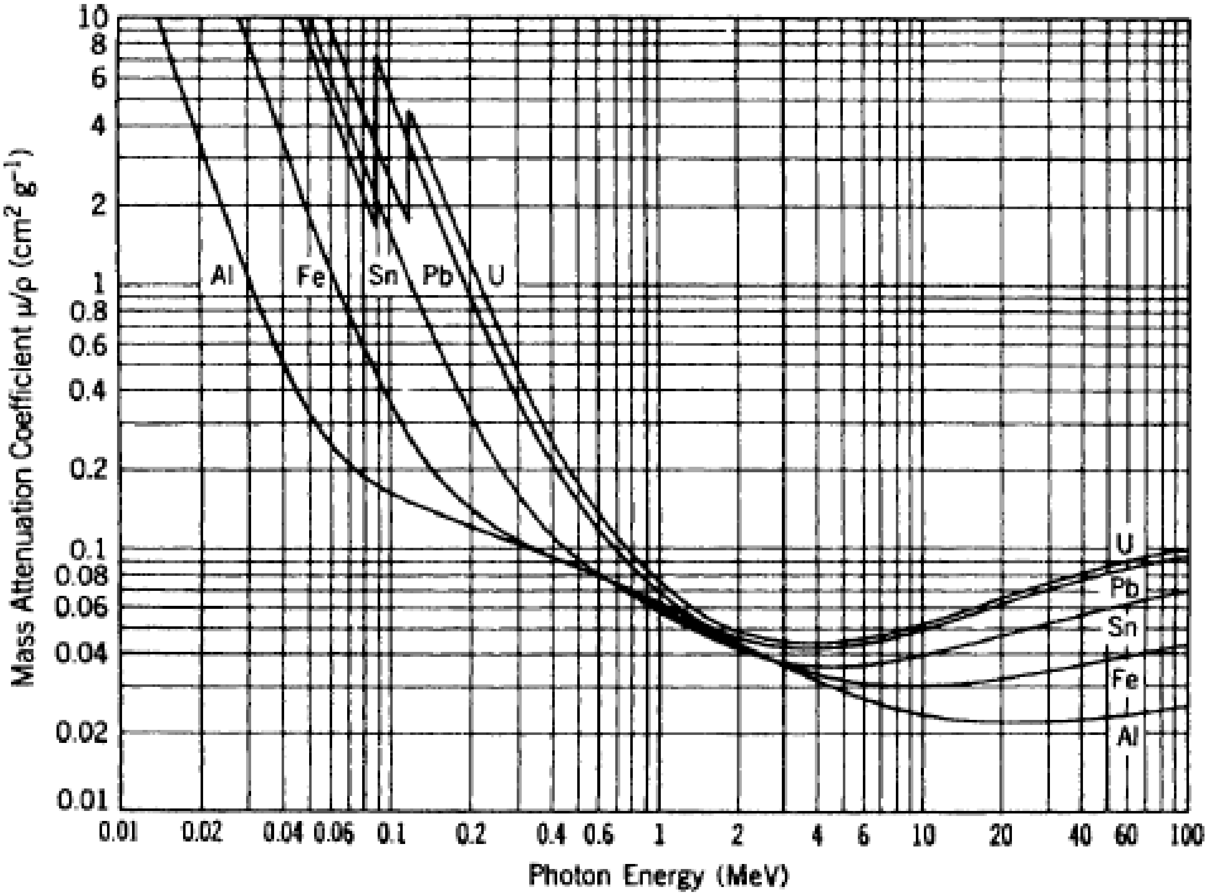
$B(\mu x) = Ae^{-\alpha_1(\mu x)} + (1 - A)e^{-\alpha_2(\mu x)}$, where A, α_1, α_2 are functions of gamma-ray energy and the shielding material. A table for these factors can be found attached.

Assuming that the function you found in part (a) for the gamma-ray intensity for a line source is the unshielded intensity, $I_u(x)$, we will calculate the shielded gamma-ray intensity, $I_s(x)$, by the following function using the buildup factor and attenuation coefficient for lead:

$I_s(x) = I_u(x)B(\mu x)e^{-\mu x}$, where μx is the number of relaxation lengths or mean free paths for the thickness of the lead shielding to be used. The density of lead is 11.34 g/cm^3. Here we will use the Taylor form of the buildup factor $B(\mu x)$. For this purpose we will only be concerned with the highest energy 2.75 MeV gamma-rays.

If we need to use 35 cm of lead around the liquid sodium coolant pipe in order to reduce the exposure rate to 2.5 mR/hr at 1 meter from the pipe surface, what would the unshielded exposure rate in R/hr be at that point?

Hint: first calculate the reduction in the exposure rate due to 35 cm of lead shielding, then use that result to determine the unshielded exposure rate.


Mass Attenuation Coefficient μ/ρ (cm² g⁻¹)
Photon Energy (MeV)
Al
Fe
Sn
Pb
U
10
8
6
4
2
1
0.8
0.6
0.4
0.2
0.1
0.08
0.06
0.04
0.02
0.01
0.01 0.02 0.04 0.06 0.1 0.2 0.4 0.6 1 2 4 6 10 20 40 60 100

(5) Advanced Engineering Mathematics

a) (35%) What is the *method of Lagrange* multipliers?

b) (65%) The geometric buckling for a parallelepiped reactor of size $(a * b * c)$ is given by

$$B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$$

If the materials composition is fixed, use the *Lagrange multipliers method* to derive a relation between *a*, *b*, and *c* for achieving a minimum reactor volume.