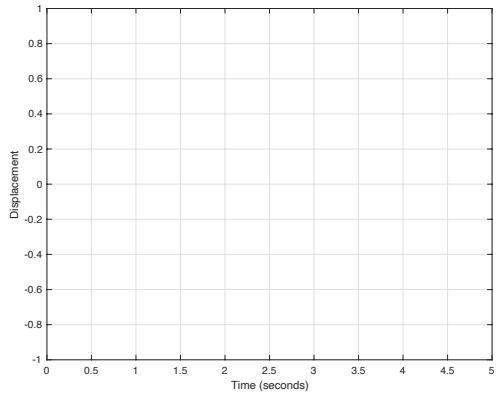
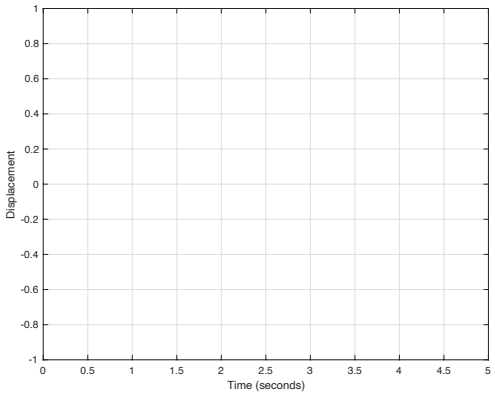
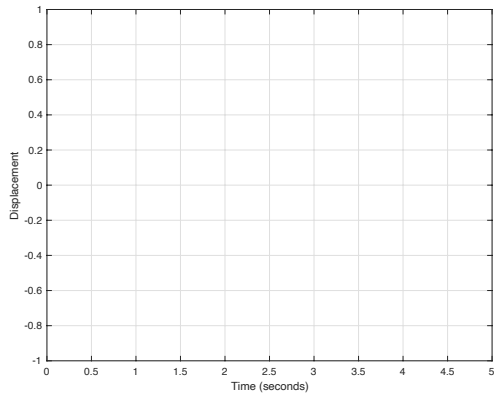
Vibrations - 1

Question 1: (15 pts)

- Sketch the characteristic response of an undamped, underdamped and overdamped system to initial conditions x_0 =1 and v_0 =0? Assume the natural frequency is 2π for all cases. Note that damping ratio is not given thus no exact decay rate is expected where it may apply.

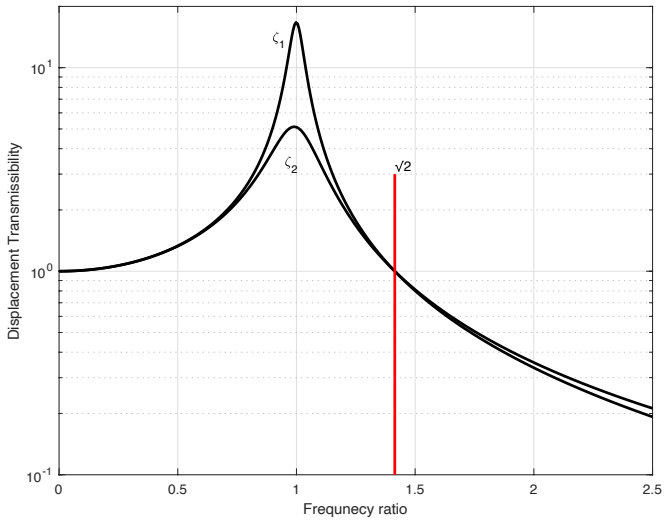






Question 2: (10 pts)

- In a base excitation problem, the displacement transmissibility (X/Y) is plotted for two cases of damping ζ_1 and ζ_2 , where ζ_2 has a higher damping value. Which is the curve belonging to ζ_1 and respectively ζ_2 , past $r = \sqrt{2}$? Mark on the graph with an arrow and label for ζ_1 and ζ_2.



Problems

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Question 3: (15 pts) Given the following system $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 \cos \omega t$, at resonance:

The input force is in phase with (5pts):

1) Displacement
2) Velocity
3) Acceleration

Very simply show how you know this. (10pts)

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Question 4: (10 pts)
Place $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 \cos \omega t$ in first order form and in a matrix format as $\dot{x}(t) = Ax(t) + f(t)$

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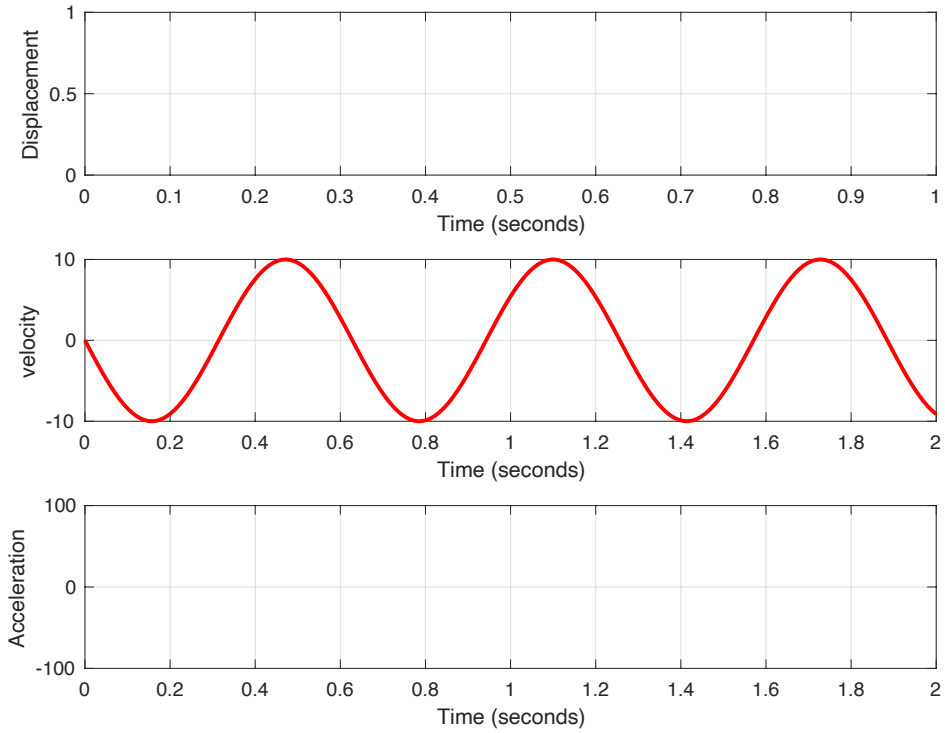
Question 5: (10 pts)
Given the response below to a single degree of freedom system with no damping, complete the other two cases labeled on the figures. Assume the natural frequency is 10rad/sec.



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Question 6: (10 pts)

What is the particular solution of $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 \cos \omega t$?

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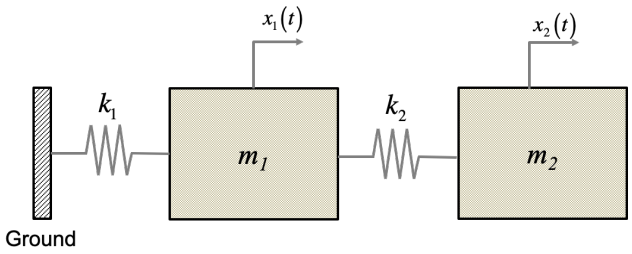
Question 7: (20 pts)

The system below can be described as $M\ddot{\boldsymbol{x}}(t) + K\boldsymbol{x}(t) = 0$ in matrix form, where M and K are square matrices.

1- How many frequencies will this system have if it represents the image to the right? (5pts)
2- If the solution can be expressed as $\boldsymbol{x}(t) = \mathbf{u}e^{j\omega t}$ show how you arrive at the eigenvalue problem. (15pts)

Note: There is no need to derive the equation of motion



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Question 8: (10 pts)

Apply the Laplace transform to $m\ddot{x} + c\dot{x} + kx = F(t)$ given zero initial conditions and arrive at the transfer function ($H(s) = X(s)/F(s)$). Show how the transfer function and the frequency response function are related.