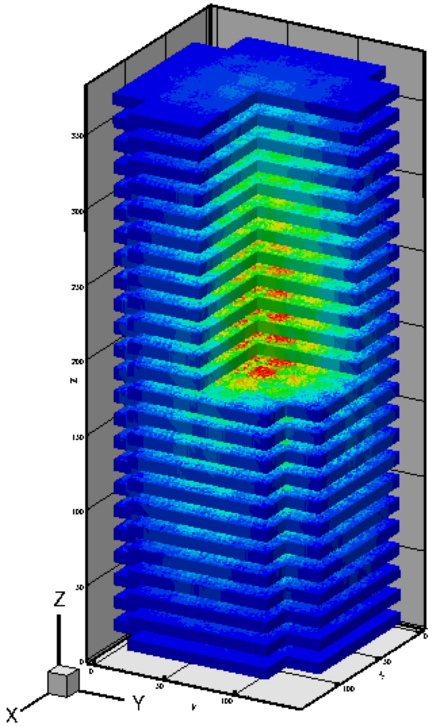
Evaluation of RAPID for a UNF cask benchmark problem



Valerio Mascolino, Alireza Haghighat, and Nathan Roskoff
Virginia Tech
Nuclear Science and Engineering Laboratory
Nuclear Engineering Program, Mechanical Engineering Department
Arlington, VA
http://nsel.ncr.vt.edu

Outline

- Purpose
- The RAPID code system
- GBC-32 cask computational benchmark
  - System description
  - Establishment of MCNP reference models
  - Comparison of RAPID to MCNP reference models
- Concluding remarks and future work

Purpose

▷ Benchmarking of the RAPID Multi-stage Response-function Transport (MRT) code system against the GBC-32 Cask system through comparison with MCNP reference models.

The RAPID (Real-time Analysis for Particle transport and In-situ Detection) code system

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Virginia Tech Research Center, Arlington, VA

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Development of Transport Formulations for Real-Time Applications

▷ The RAPID code system is developed based on the MRT (*Multi-stage Response-function Transport*) methodology; the MRT methodology is described as follows:

1. *Partition a problem into stages*
2. *Represent each stage by a response function or set of response coefficients*
3. *Pre-calculate response functions and/or coefficients (one time)*
4. *Couple stages through a set of linear system of equations*
5. *Solve the linear system of equations iteratively in real-time*

The RAPID Code System

- RAPID is capable of calculating the system *eigenvalue* k_{eff}, *3D (pin-wise & axially-dependent) fission density distribution, and detector response.*

- RAPID is comprised of **five stages**:

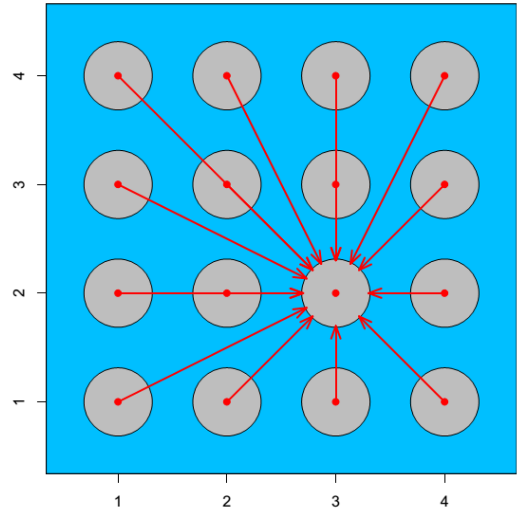
  Stage 1: Calculation of fission matrix (FM) coefficients, and generation of a database
  Stage 2: Calculation detector field-of-view (FOV) and importance function database
  Stage 3: Processing of FM coefficients
  Stage 4: Solving a linear system of equations, i.e., **Fission Matrix (FM) formulation**
  Stage 5: Detector response calculation

Fission Matrix (FM) formulation

Eigenvalue formulation

$$S_i = \frac{1}{k}\sum_{j=1}^{N} a_{i,j} S_j$$

k is eigenvalue
S_j is fission source
$a_{i,j}$ is the number of fission neutrons produced in cell i due to a fission neutron born in cell j.



Subcritical multiplication formulation

$$S_i = \sum_{j=1}^{N} (a_{i,j} S_j + b_{i,j} S_j^{intrinsic})$$

$b_{i,j}$ is the number of fission neutrons produced in cell i due to a source neutron born in cell j.

The GBC-32 cask computational benchmark

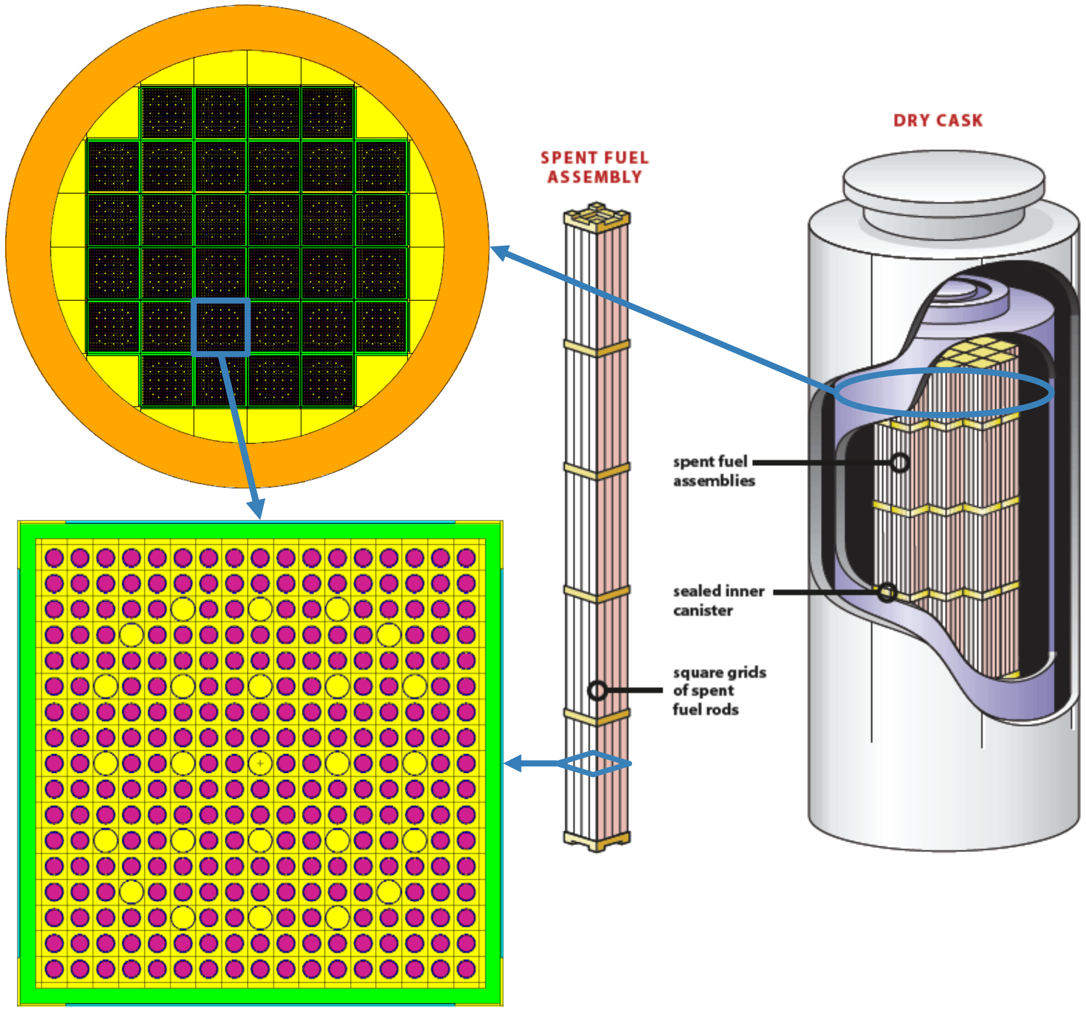
GBC-32 cask computational benchmark

- **Geometry**

32 Fuel assemblies
Stainless steel (SS304) cylindrical canister
Inter-assembly Boral absorber panels
Height of the canister: 470.76 cm

- **Fuel assembly**

17x17 Optimized Fuel Assembly (OFA)
25 instrumentation guides
<u>Fresh</u> UO_2 4% wt. enriched fuel pins
Active height: 365.76 cm



Establishment of a reference MCNP model

▷ A reference MCNP model has been established for comparison with the RAPID results

▷ This has been accomplished by examining the convergence of the fission source distribution for a single-assembly model by:

- Parametric analysis of MCNP eigenvalue parameters:
  NSK - Number of Skipped Cycles (NSK)
  NAC - Number of Active Cycles (NAC)
  NPS - Number of Particles per Cycle (NPS)

- Cycle-to-cycle correlation analysis

▷ The results of the single-assembly analysis have been extended to the full cask

Known eigenvalue Monte Carlo difficulties

- **Source convergence:** Used Nuclear Fuel (UNF) pools or casks due to the presence of absorbers, suffer from undersampling that may result in a biased solution.

- **Cycle-to-cycle correlation:** previous generation is used as source in the power-iteration method, correlation may take places between successive cycles. Statistical uncertainties might be underestimated.

Techniques for examining source convergence

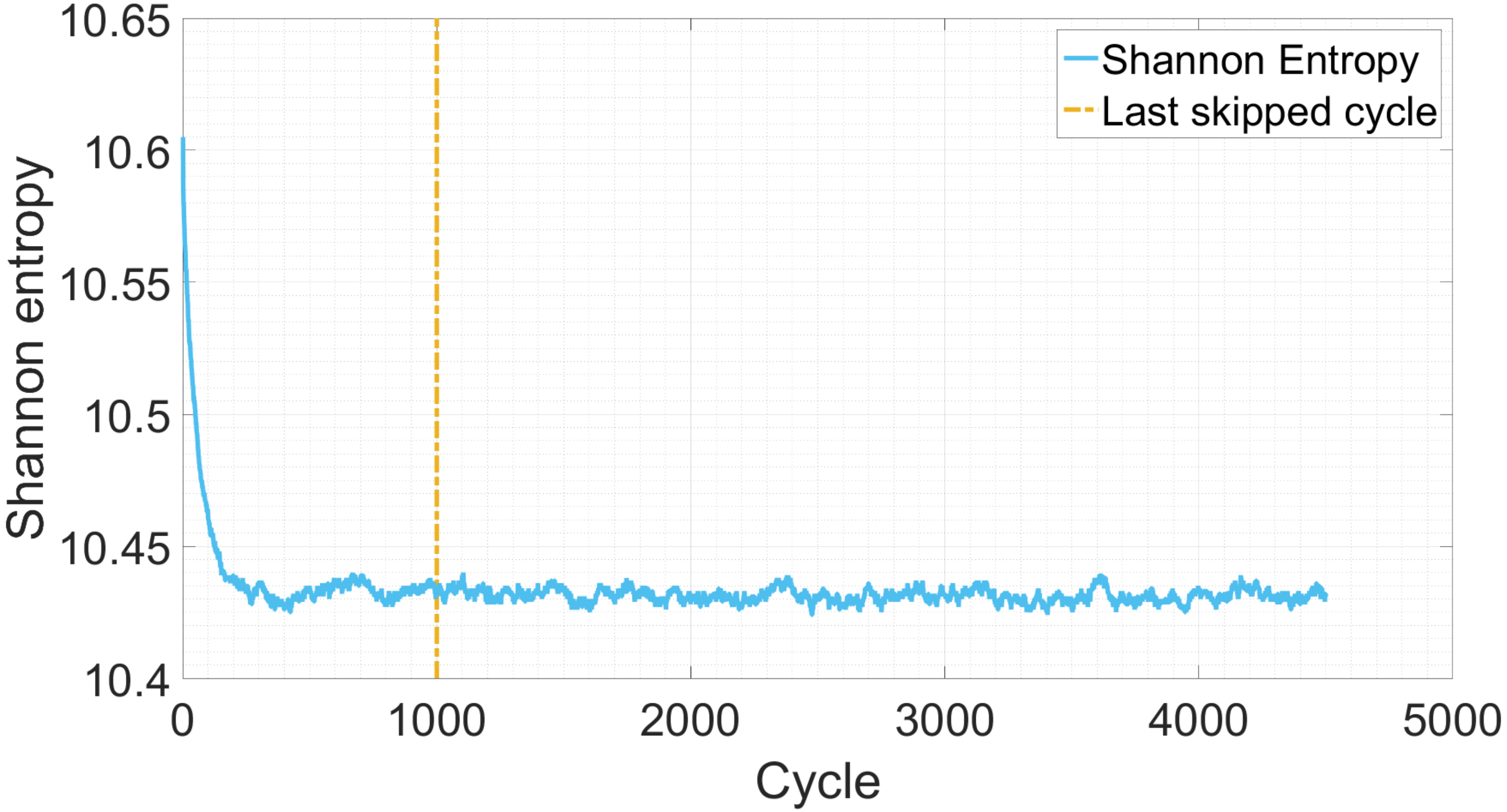
(264 pins x 24 axial nodes = 6336 tally regions)

- Relative difference
- Shannon entropy stabilization
- L_∞, L_1, and L_2 norms
- Center of Mass (COM)
- Cycle-to-cycle correlation via *replication*

NAC, NSK, and NPS parametric analyses

- Based on the single assembly model -

Shannon entropy



NSK=1000, NPS=10^6

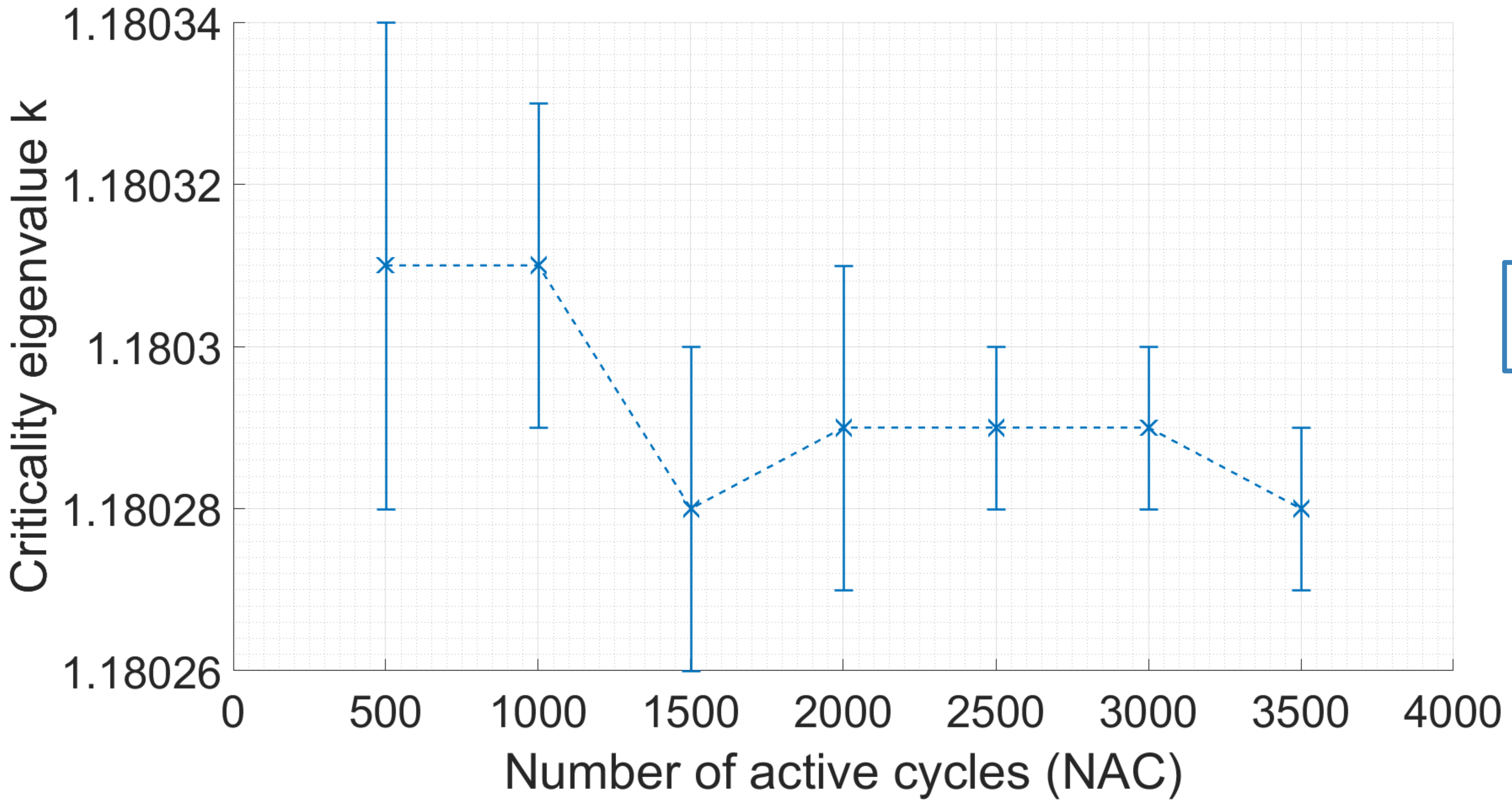
~158 *particles per tally region*

$$H = -\sum_{i=1}^{m} S_i \log_2(S_i)$$

where m is the number of subregions, and S_i is the ratio of fission neutrons newly generated in i^{th} subregion and the total number of fission neutrons in the previous generation.

k_{eff}: variation as a function NAC

NAC

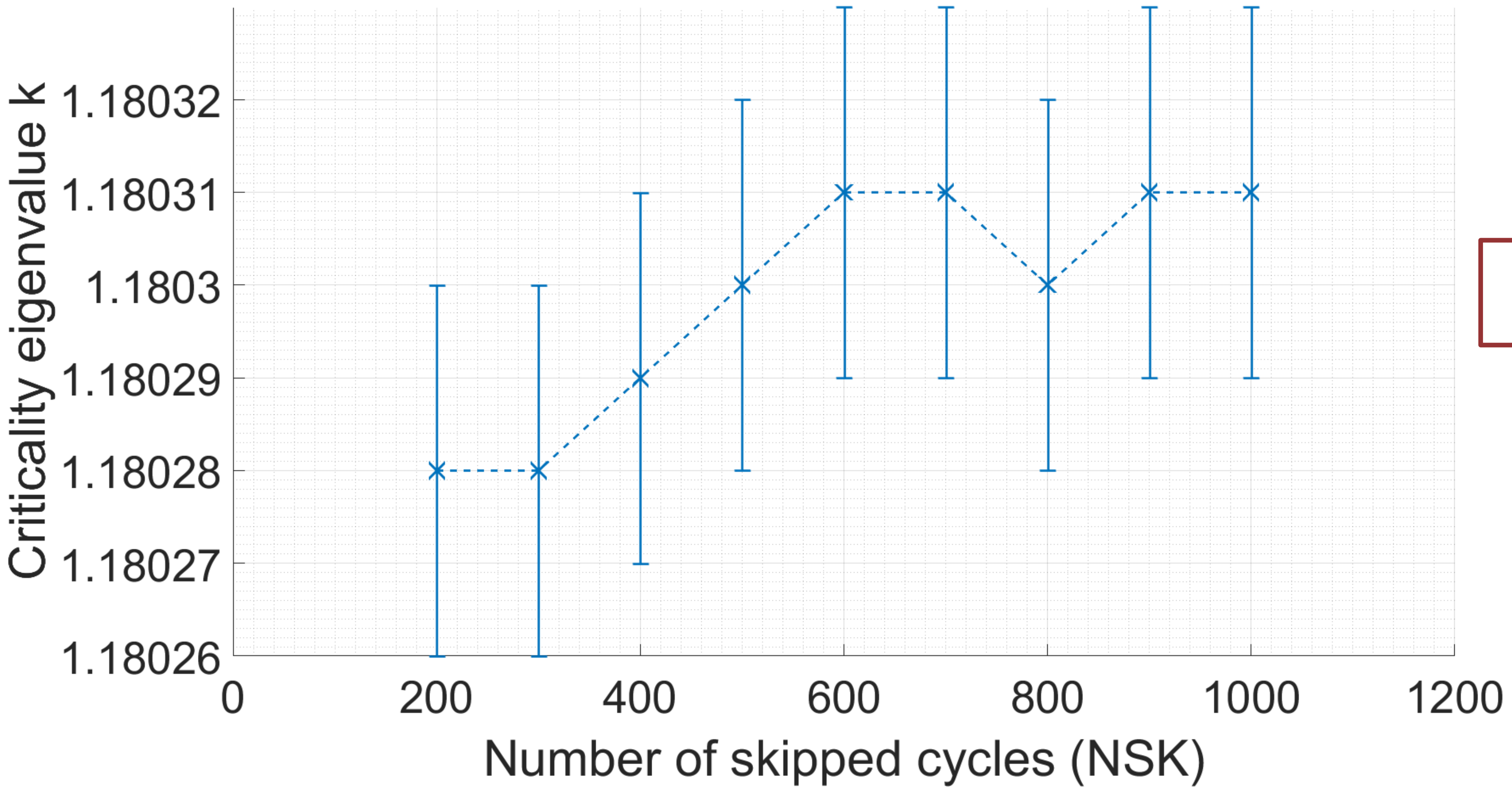


NSK=1000,
NPS=10^6

Note: eigenvalue oscillates within the MCNP calculated uncertainties

k_{eff}: variation as a function NSK

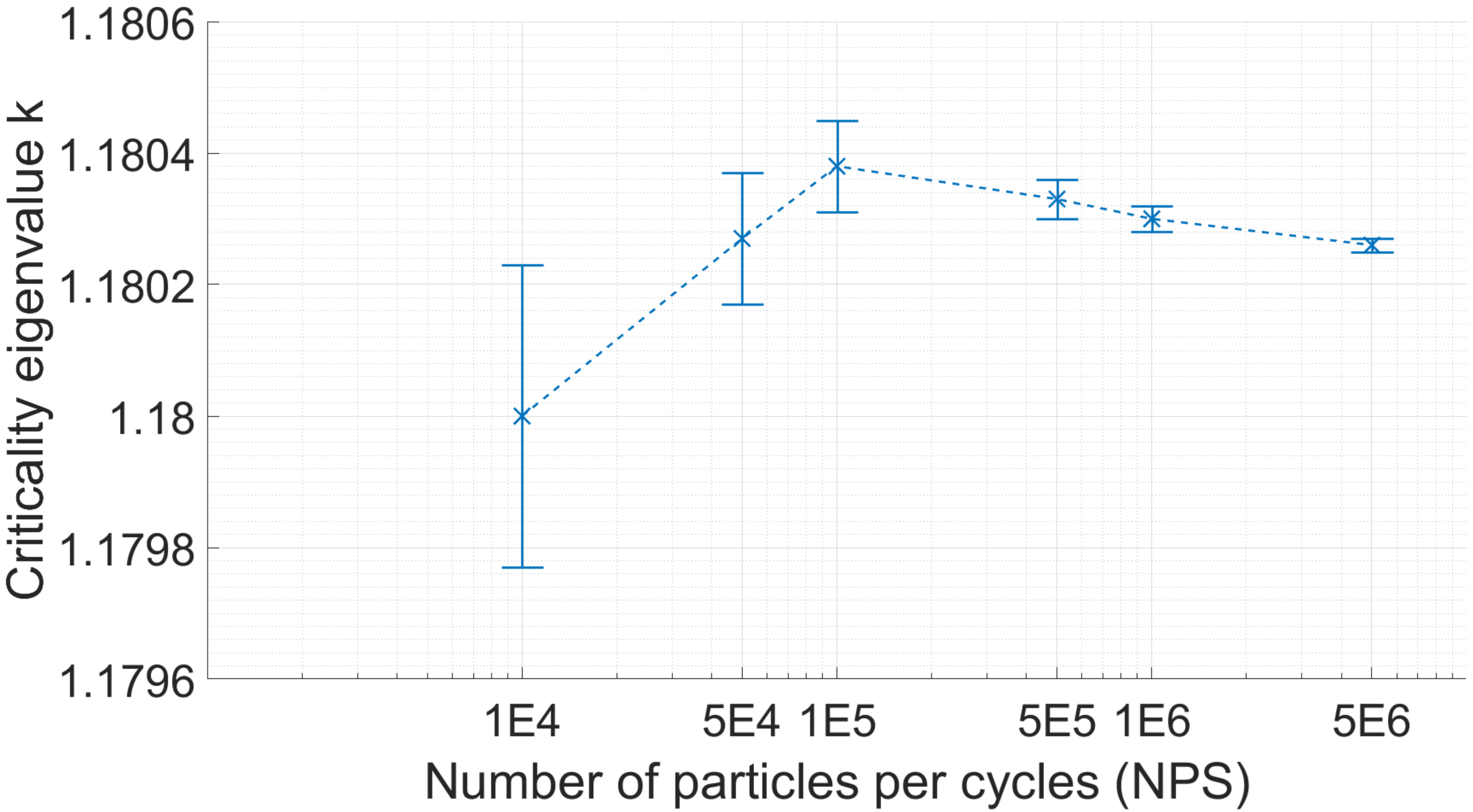
NSK



NAC=1000,
NPS=10^6

k_{eff}: variation as a function NPS

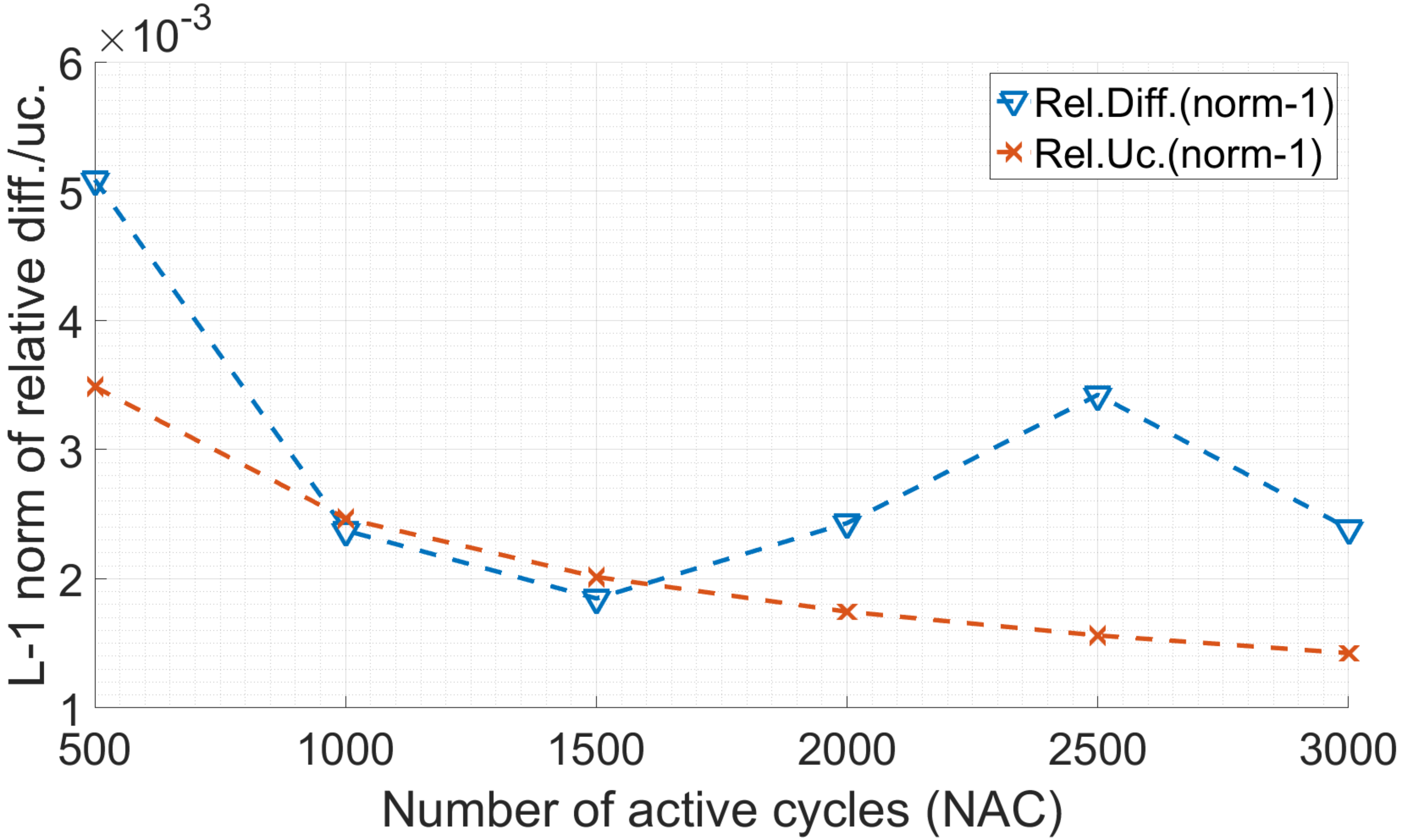
NPS



NAC=1000, NSK=500

Fission density: L_1-norms of relative differences and uncertainties

NAC



NSK=1000, NPS=10^6

$$(Rel.Diff.)_{i,n} = \frac{S_{i,n} - S_{i,md}}{S_{i,md}}$$

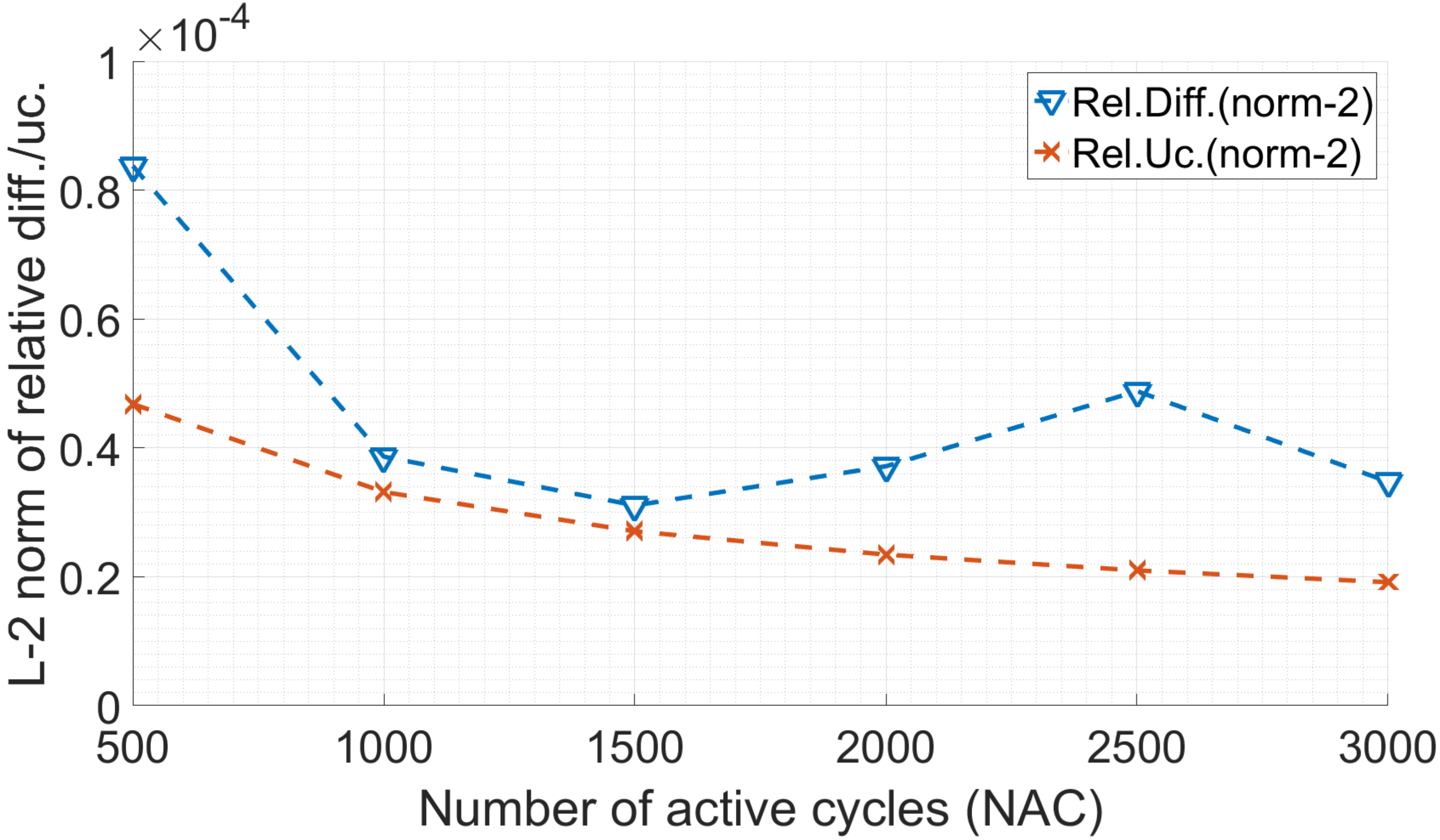
where $S_{i,n}$ and $S_{i,md}$ indicate the fission density value of the i^{th} tally for the n^{th} and the most-detailed (md) cases respectively.

$$(L_1 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}|X_i|$$

where N_t is the number of tally regions, and X is a vector containing the relative differences (blue line) or the relative uncertainties (orange line).

Fission density: L_2-norms of relative differences and uncertainties

NAC



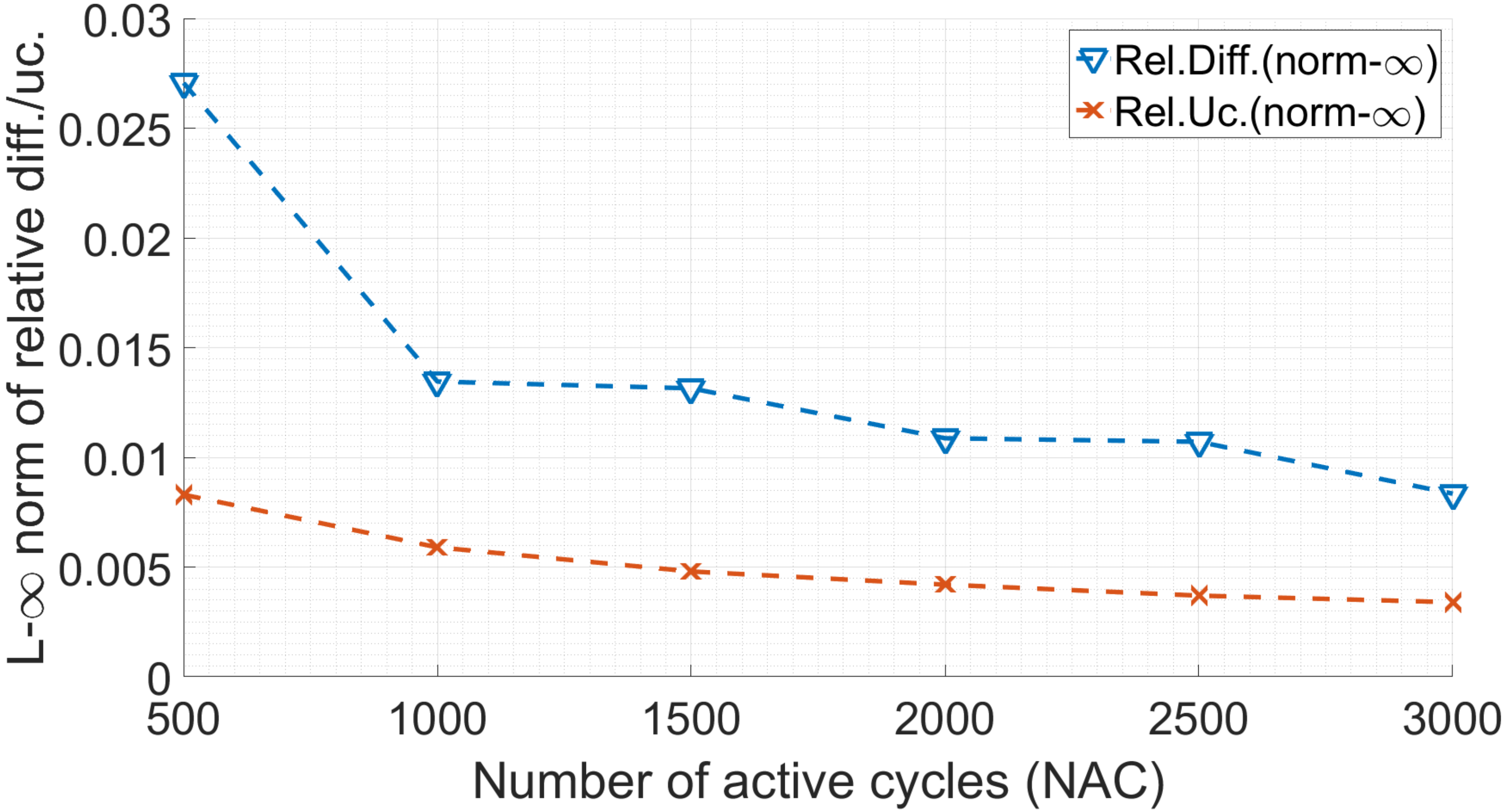
NSK=1000,
NPS=10^6

$$(L_2 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}\sqrt{(X_i - \bar{X})^2}$$

where N_t is the number of tally regions, and X is a vector containing the relative differences (blue line) or the relative uncertainties (orange line).

Fission density: L_∞-norms of relative differences and uncertainties

NAC



NSK=1000, NPS=10^6

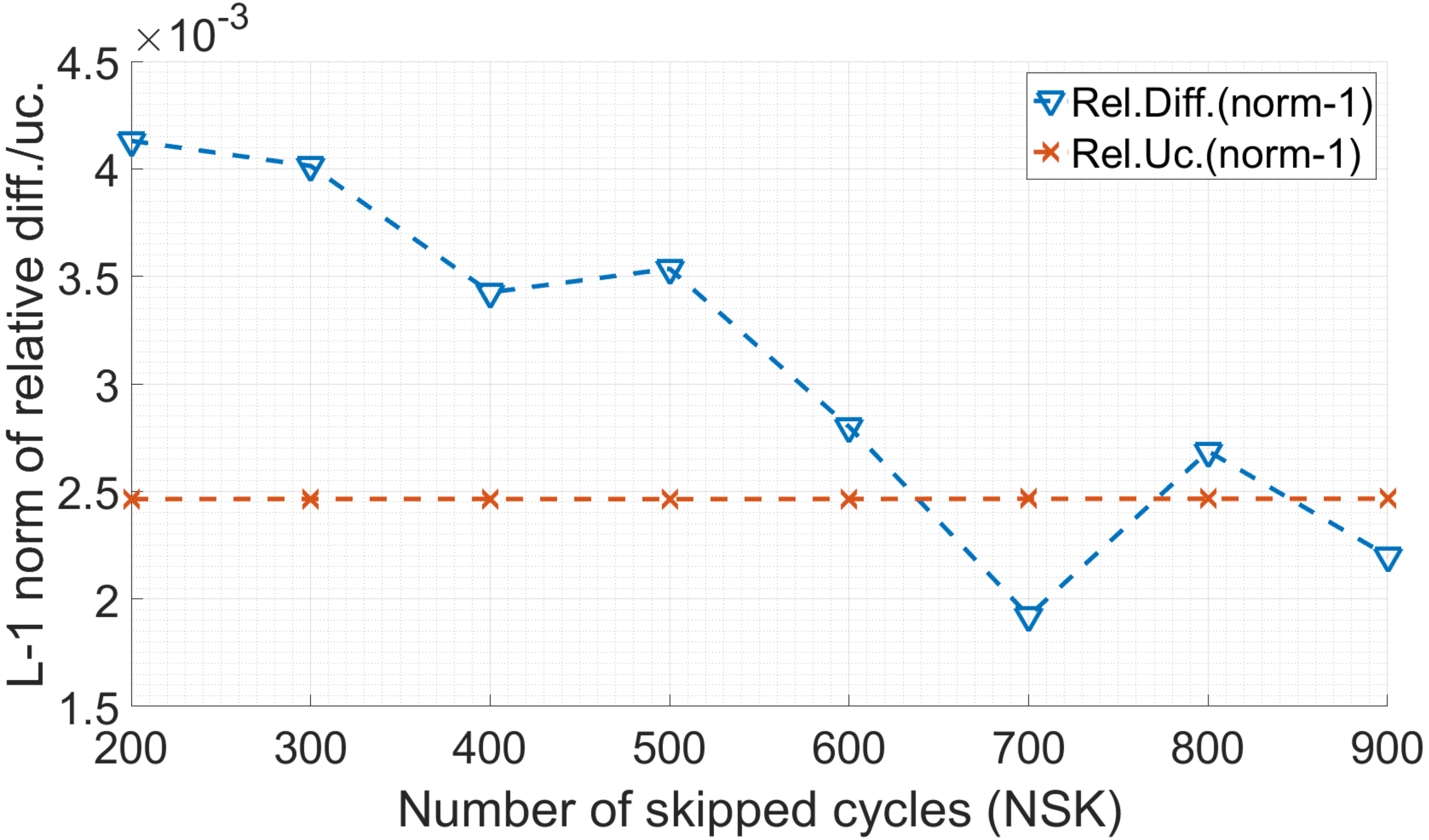
$$(L_\infty - norm)_n = \max_i S_{i,n}$$

where N_t is the number of tally regions, and X is a vector containing the relative differences (blue line) or the relative uncertainties (organe line).

Fission density: L_1-norms of relative differences and uncertainties

NSK

NAC=1000, NPS=10^6



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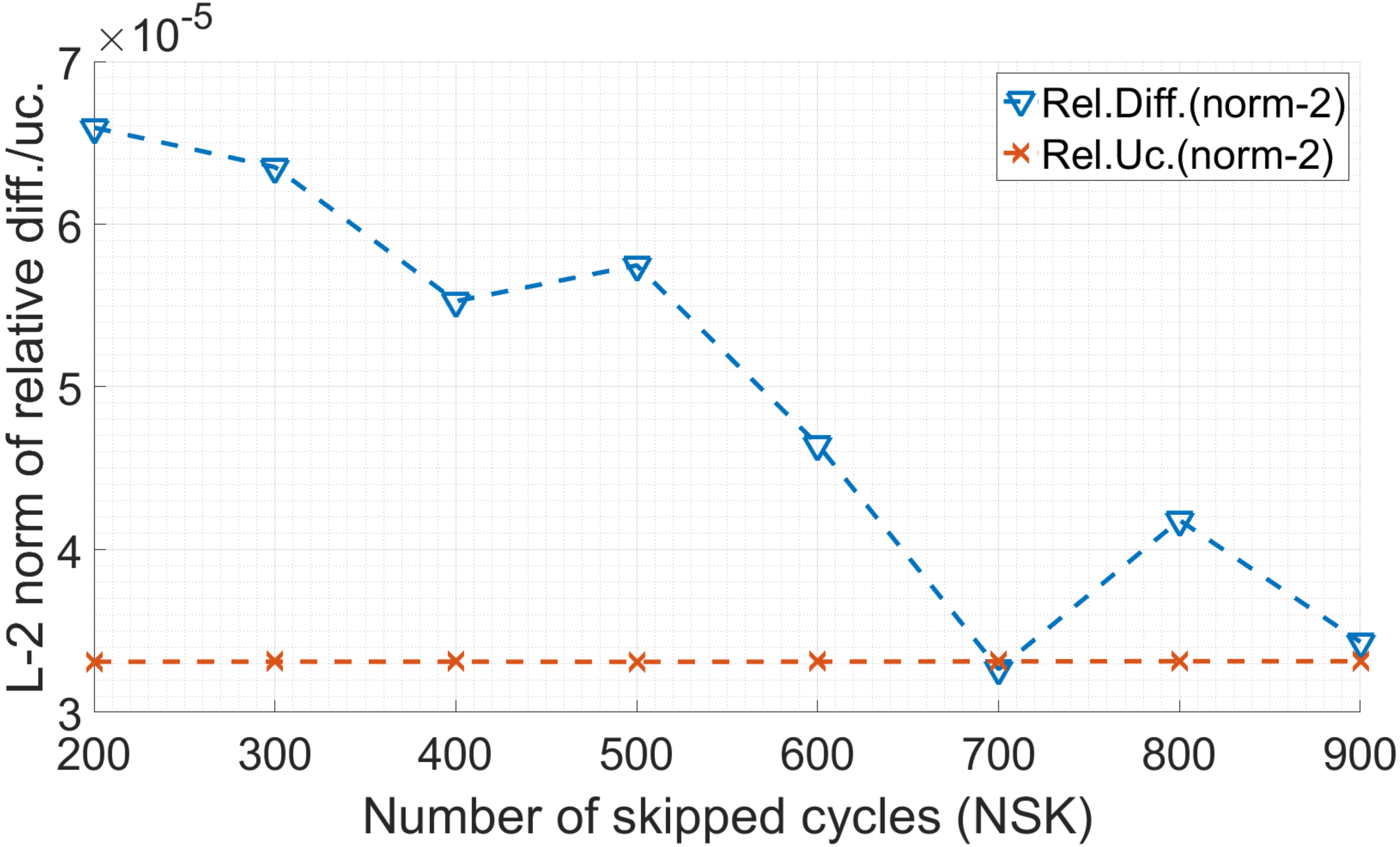
Fission density: L_2-norms of relative differences and uncertainties

NSK



NAC=1000, NPS=10^6

$$(L_2 - norm)_n = \frac{1}{N_t}\sum_{i=1}^{N_t}\sqrt{(X_i - \bar{X})^2}$$

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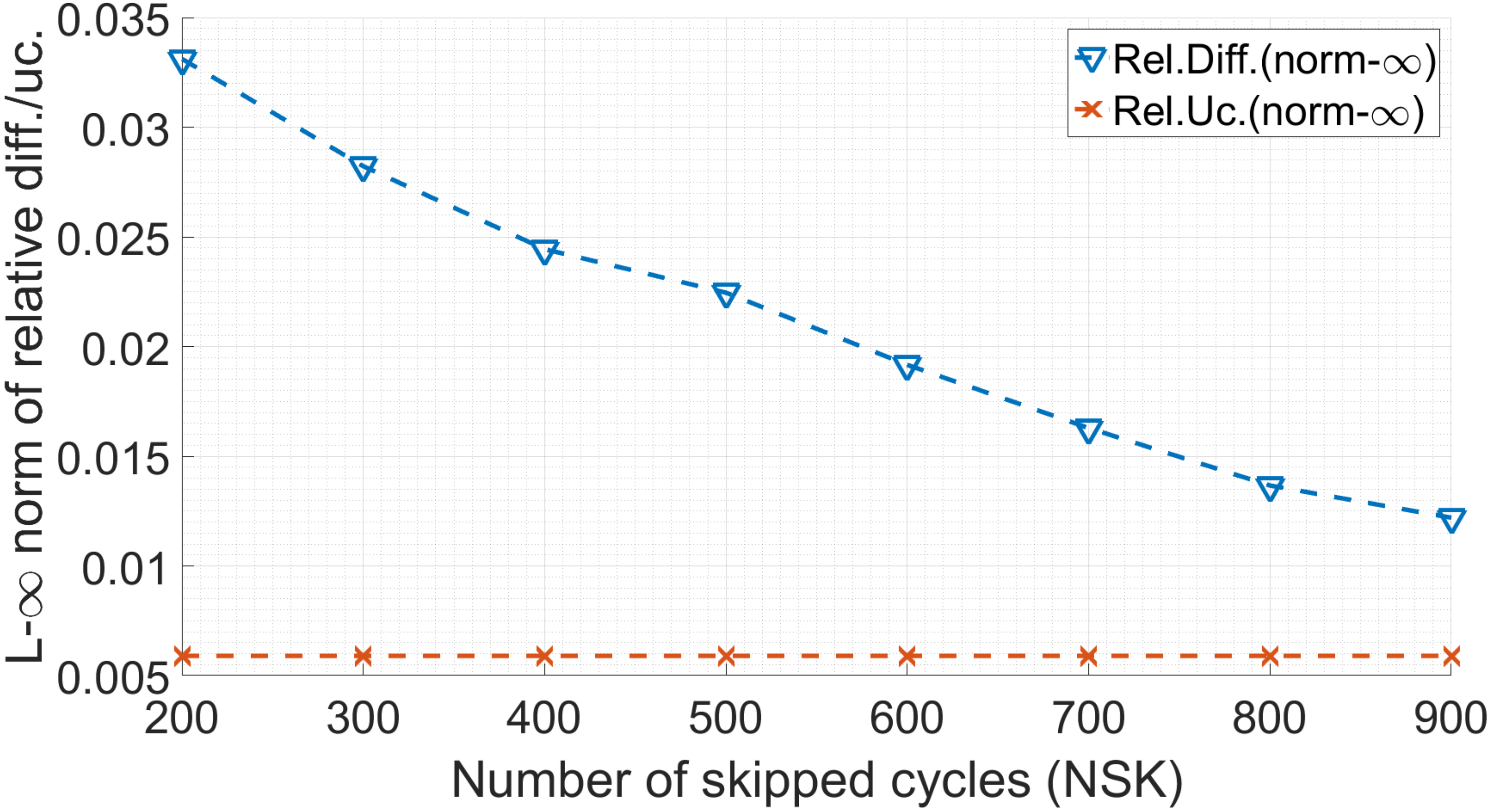
Fission density: L_∞-norms of relative differences and uncertainties

NSK



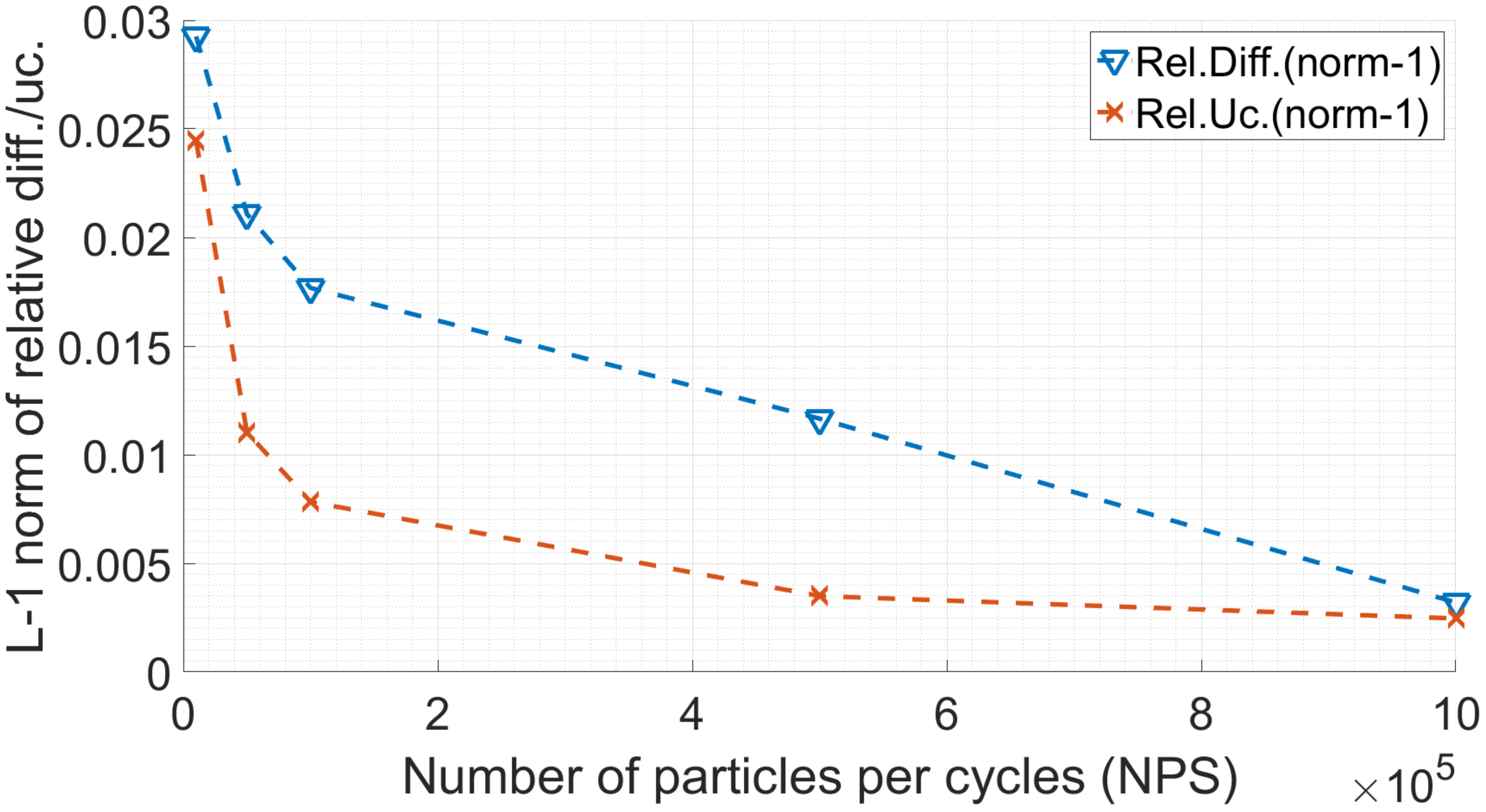
NAC=1000, NPS=10^6

$$(L_\infty - norm)_n = \max_i S_{i,n}$$

where N_t is the number of tally regions, and X is a vector containing the relative differences (blue line) or the relative uncertainties (red line).

Fission density: L_1-norms of relative differences and uncertainties

NPS



NSK=500,
NAC=1000

$$(Rel.Diff.)_{i,n} = \frac{S_{i,n} - S_{i,md}}{S_{i,md}}$$

where $S_{i,n}$ and $S_{i,md}$ indicate the fission density value of the i^{th} tally for the n^{th} and the most-detailed (md) cases respectively.

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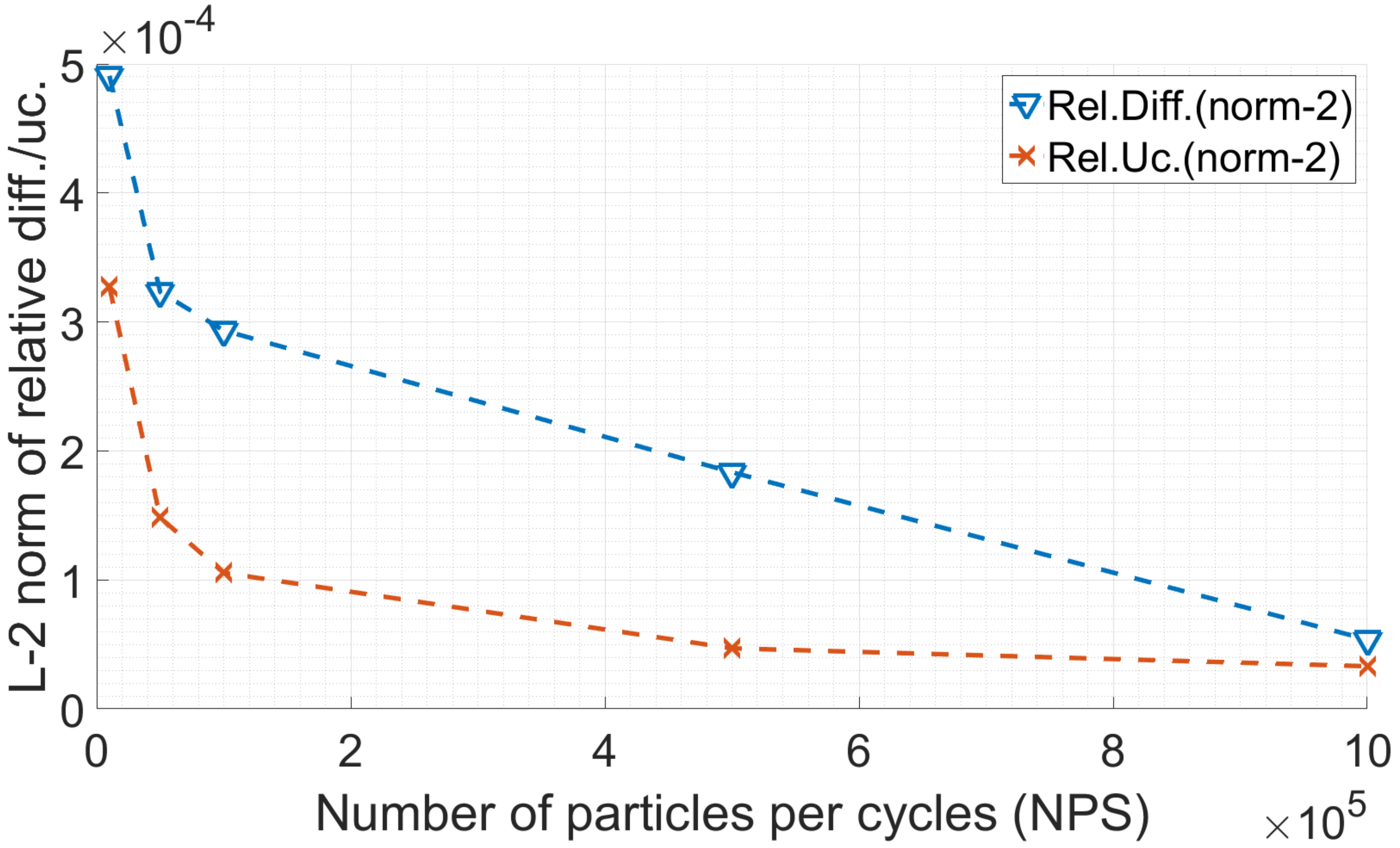
Fission density: L_2-norms of relative differences and uncertainties

NPS



NSK=500,
NAC=1000

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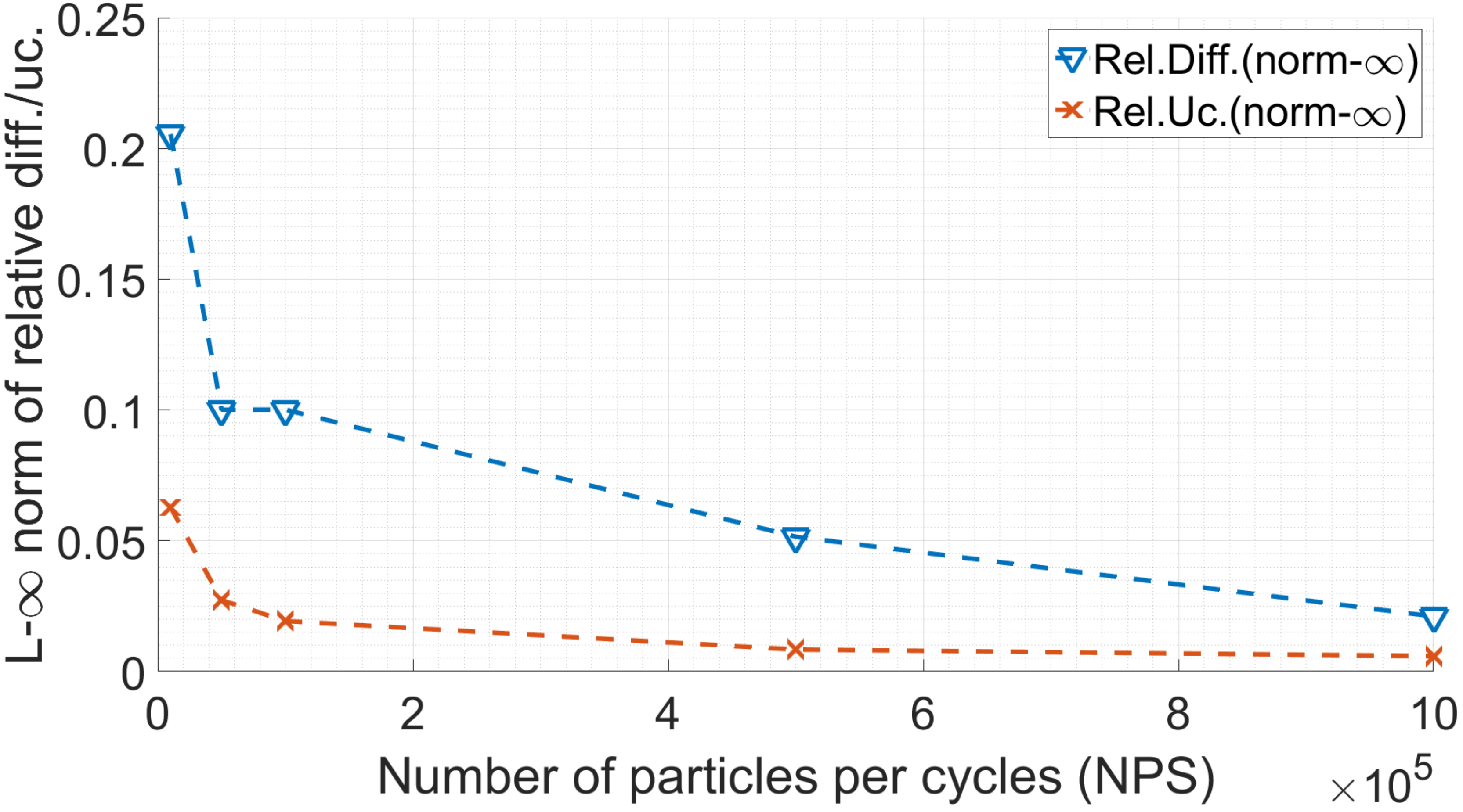
Fission density: L_∞-norms of relative differences and uncertainties

NPS



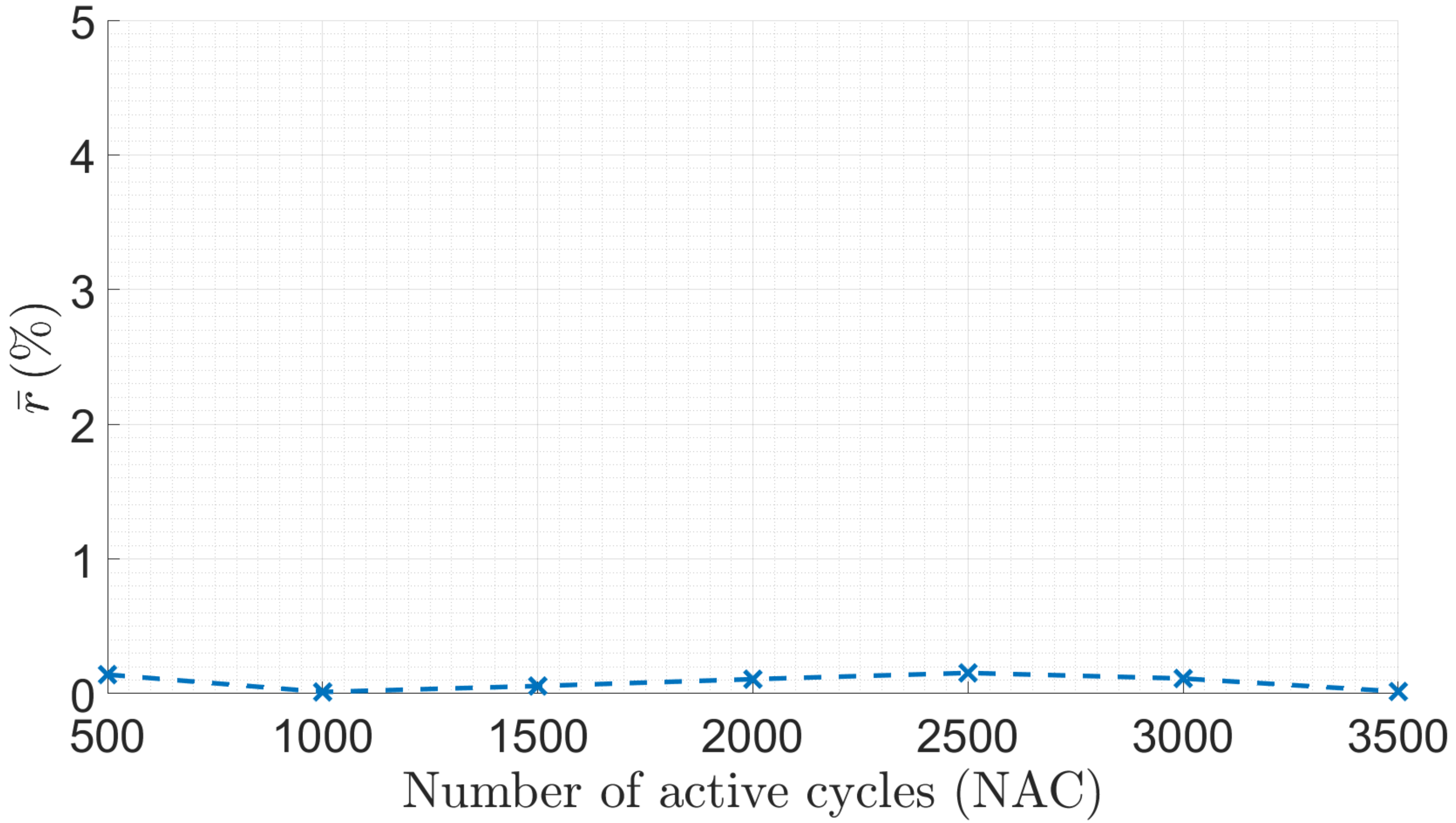
NSK=500,
NAC=1000

$$(L_\infty - norm)_n = \max_i S_{i,n}$$

where N_t is the number of tally regions, and X is a vector containing the relative differences (blue line) or the relative uncertainties (red line).

Fission density: COM distance from geometric center as a function of NAC

NAC



NSK=1000,
NPS=10^6

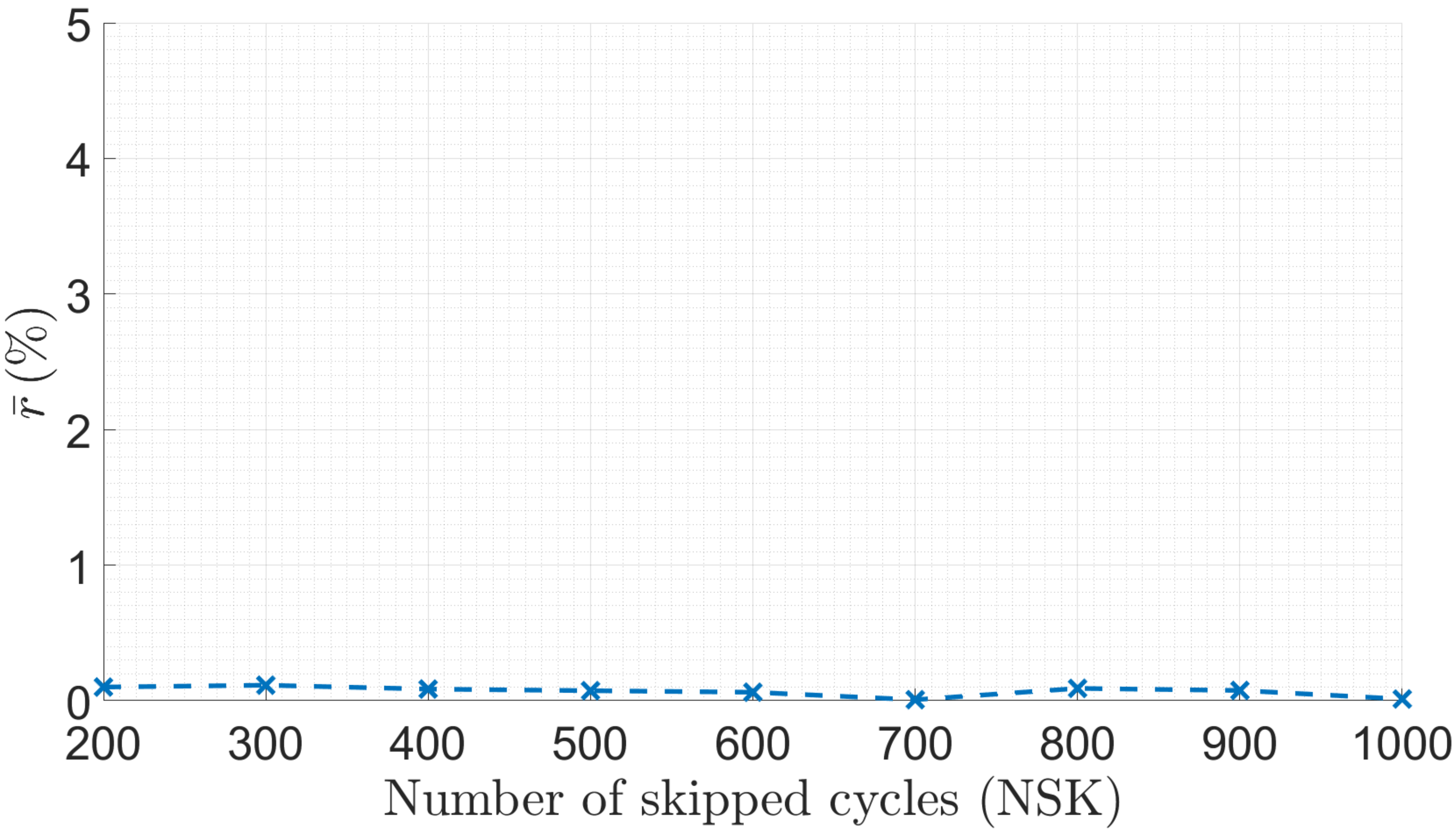
$$\overline{r_n}(\%) = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where r_i is the distance of the i^{th} region from the center of the assembly, and H is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

Fission density: COM distance from geometric center as a function of NSK

NSK



NAC=1000, NPS=10^6

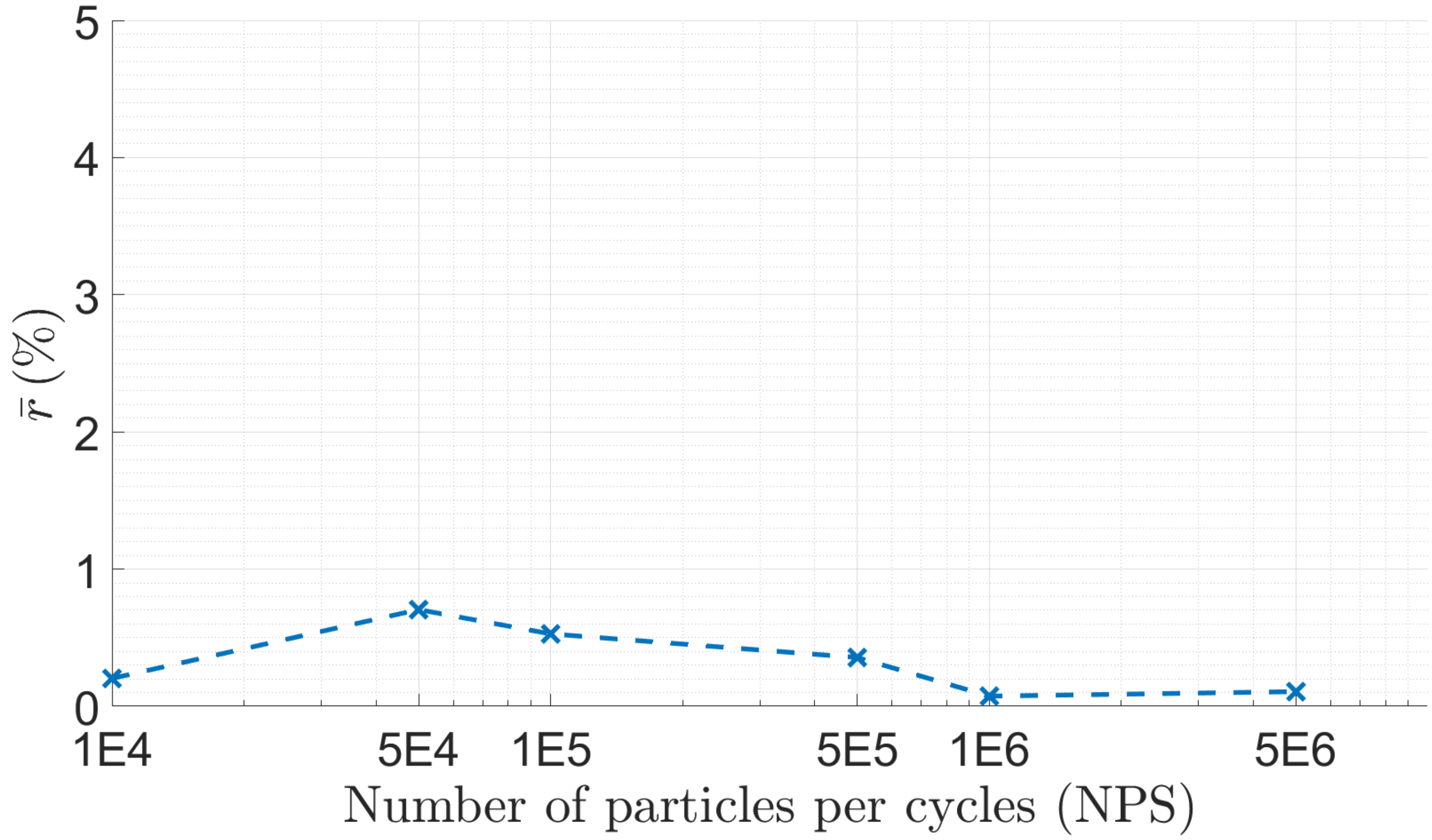
$$\bar{r}_n = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where r_i is the distance of the i^{th} region from the center of the assembly, and H is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

Fission density: COM distance from geometric center as a function of NPS

NPS



NSK=500,
NAC=1000

$$\bar{r_n} = \frac{100}{H/2 \cdot \sum_{i=1}^{N_t} S_{i,n}} \sum_{i=1}^{N_t} r_i S_{i,n}$$

where r_i is the distance of the i^{th} region from the center of the assembly, and H is the active height of the fuel.

- The COM behaves like the neutron source has **converged**

Discussion of parametric analyses results

NSK
NAC
NPS

- Shannon entropy, COM, L_1, L_2, and L_∞ behave similarly for all the parameters.

- From L_1, L_2, and L_∞ norms
  it is concluded that **relative differences *on average* are higher than the statistical uncertainties.**

- From *COM* and *Shannon entropy*,
  it is conlcuded that the **fission source has converged**

- Questions?
  Is it **possible that the statistical uncertainties are underestimated?**
  Is it caused by the **cycle-to-cycle correlation**?

Analysis of cycle-to-cycle correlation

- N_r=50 replications of a MCNP run with NSK=300, NAC=500, and NPS=10^6 are performed.

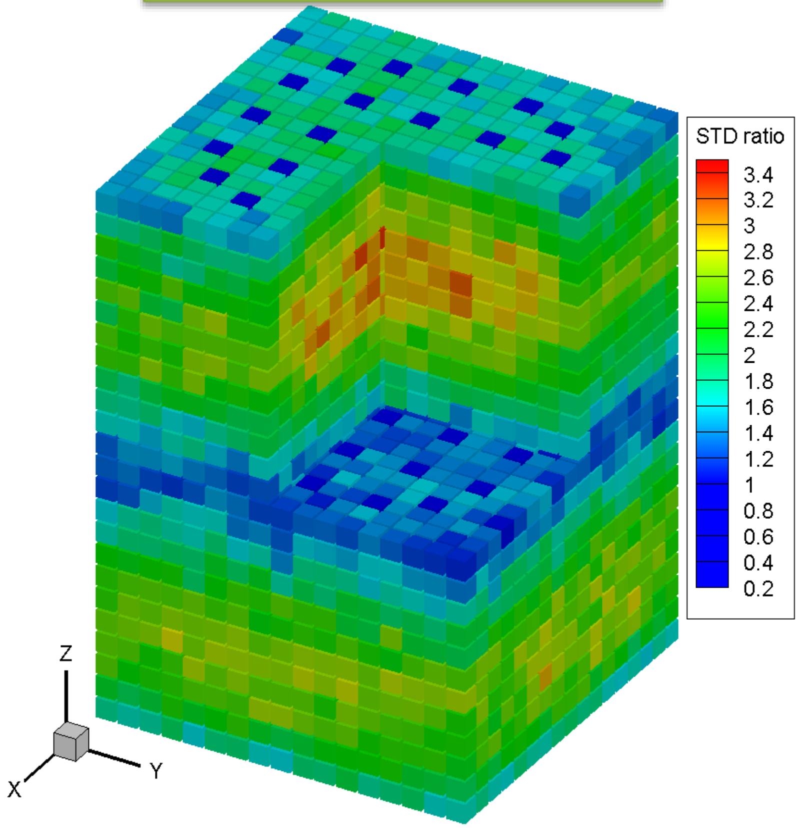
- Calculated the ratios of "actual" to MCNP statistical uncertainties, i.e., $f_{\sigma,i} = \frac{\sigma_{actual}}{\sigma_{MCNP}} = \frac{\sqrt{\frac{1}{N_r-1}\sum_{j=1}^{N_r}(S_{i,j}-\bar{S}_i)^2}}{\sigma_{MCNP}}$

- MCNP significantly **underpredicts uncertainties.**

- The weighted average of f_σ is

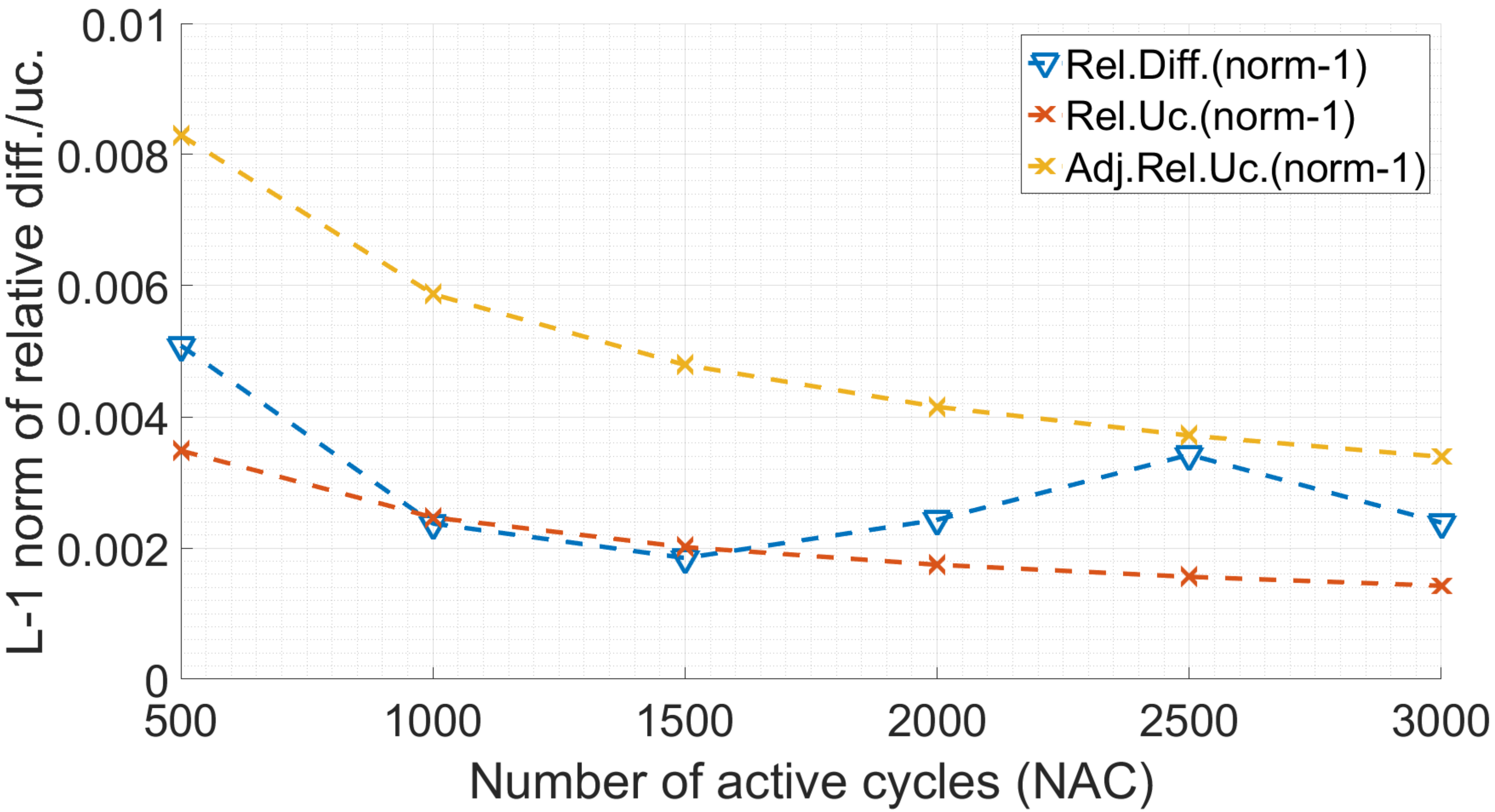
$$f_{\sigma,wgt} = \frac{\sum_{i=1}^{n} f_{\sigma i} S_i}{\sum_{i=1}^{n} S_i} = 2.28$$

f_σ distribution



$f_{\sigma,wgt}$ adjusted norms analysis

NAC

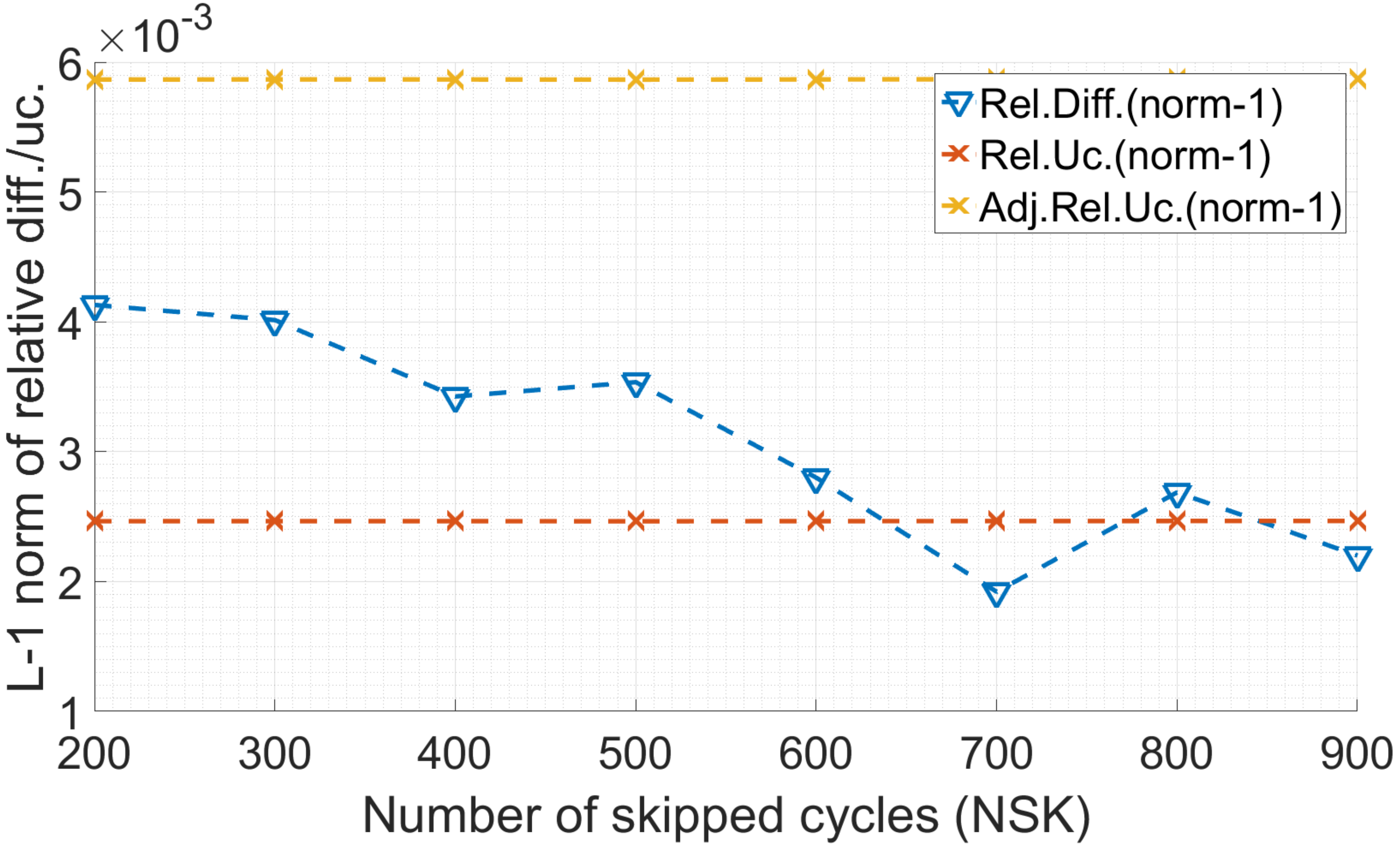


NSK=1000, NPS=10^6

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

$f_{\sigma,wgt}$ adjusted norms analysis

NSK

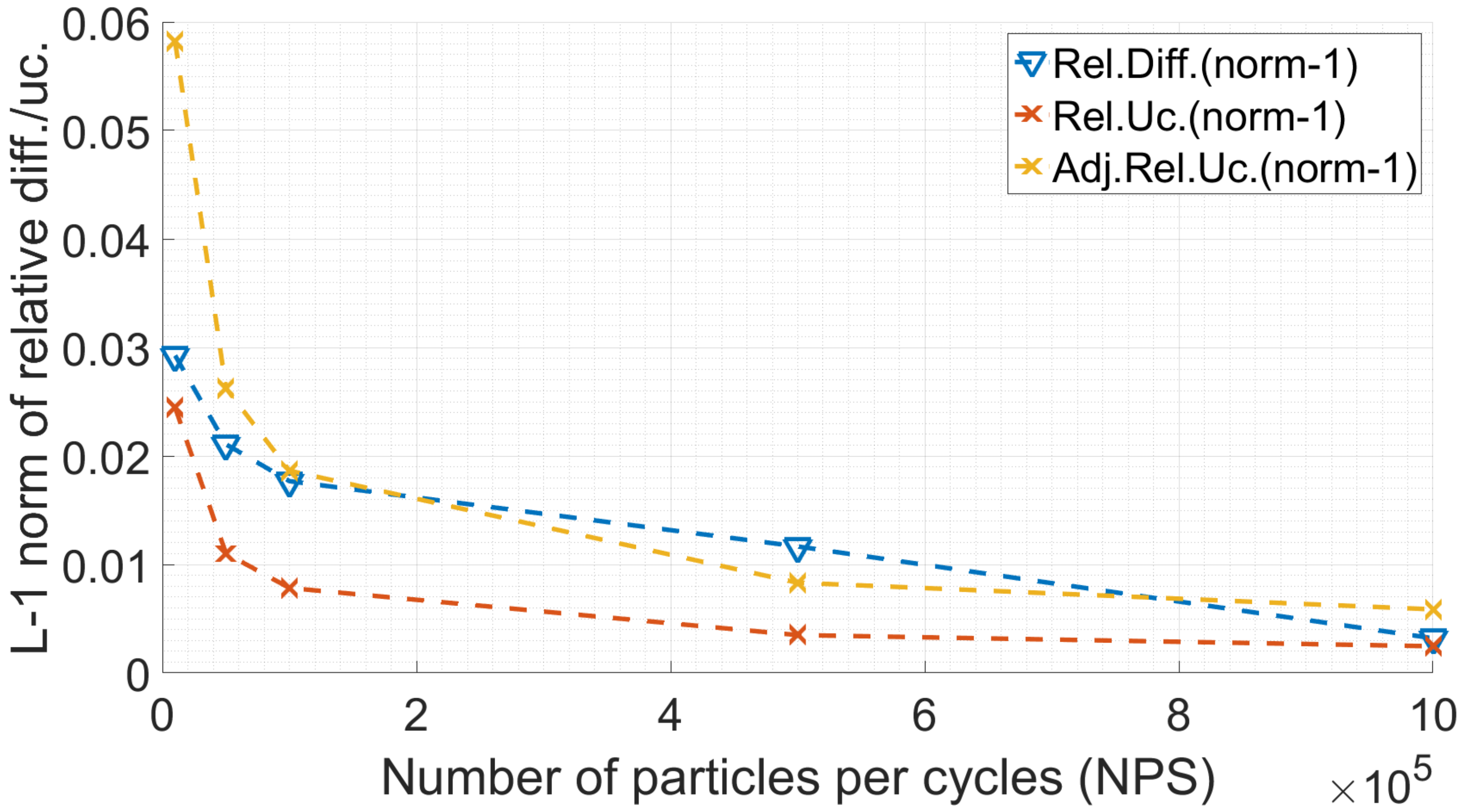


NAC=1000, NPS=10^6

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

$f_{\sigma,wgt}$ adjusted norms analysis

NPS



NSK=500,
NAC=1000

Statistical uncertainties used to calculate norms factor in the $f_{\sigma,wgt}$ correction due to cycle-to-cycle correlation.

MCNP reference Single assembly – eigenvalue parameters

- Based on this study, we selected:

  NSK=500, NAC=1000, and NPS=10^6

- This set was chosen for achieving

  relative statistical uncertainties < 1%

  for fission density tallies

MCNP Full-cask model – Eigenvalue parameters

▷ **NAC and NSK are kept constant** due to the assemblies' uncoupling caused by absorber panels.

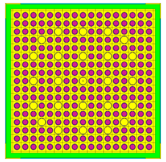
▷ NPS should be scaled by a factor of 32 , but NPS$=32 \cdot 10^6$ is computationally prohibitive.

▷Therefore, we have used a reasonable NPS of 10^5 per assembly, **i.e., $\mathbf{3.2 \cdot 10^6}$ for the full cask.**

Comparison of RAPID to MCNP reference models

- Single assembly & full cask models -

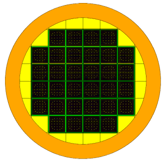
RAPID vs. MCNP – Single assembly model



- RAPID calculated and MCNP system eigenvalue (k_{eff}) and pin-wise, axially-dependent fission density distribution, i.e, **6,336 tallies**, are compared.
- **Significant speedup** is obtained using RAPID on just a single computer core.

Case	MCNP	RAPID
k_{eff}	1.18030 (± 2 pcm)	1.18092
k_{eff} relative difference	-	53 pcm
Fiss. density adjusted rel. uncertainty	0.48%	-
Fission density relative diff.	-	0.65%
Computer	16 cores	1 core
Time	666 min (11.1 hours)	0.1 min (6 seconds)
Speedup	-	6,666

RAPID vs. MCNP – Full cask model

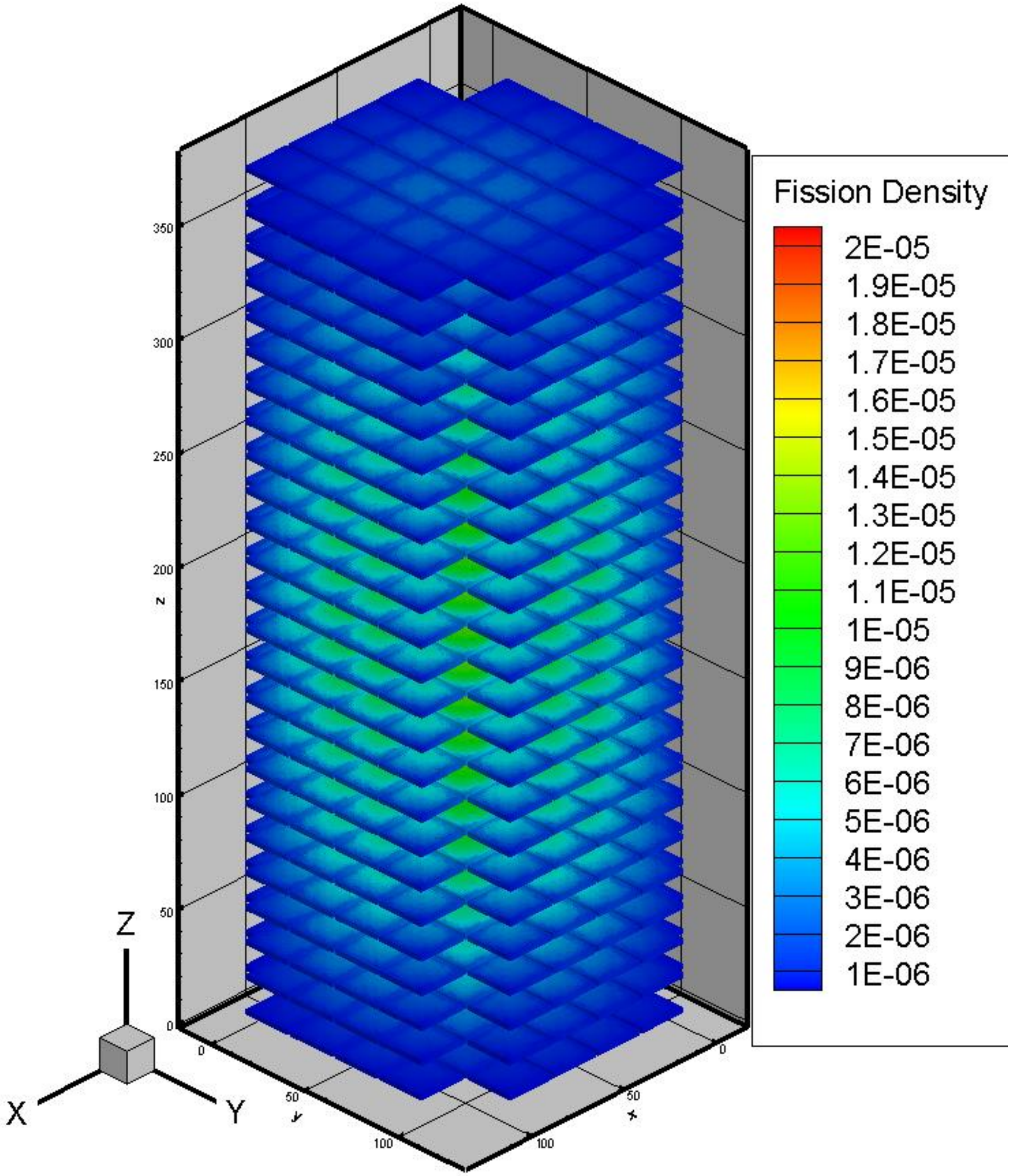


- RAPID calculated and MCNP system eigenvalue (k_{eff}) and pin-wise, axially-dependent fission density distribution, i..e, **202,752 tallies** (for ~15.8 particles per tally region) , are compared.

- **The speedup increases with the dimension of the model.**

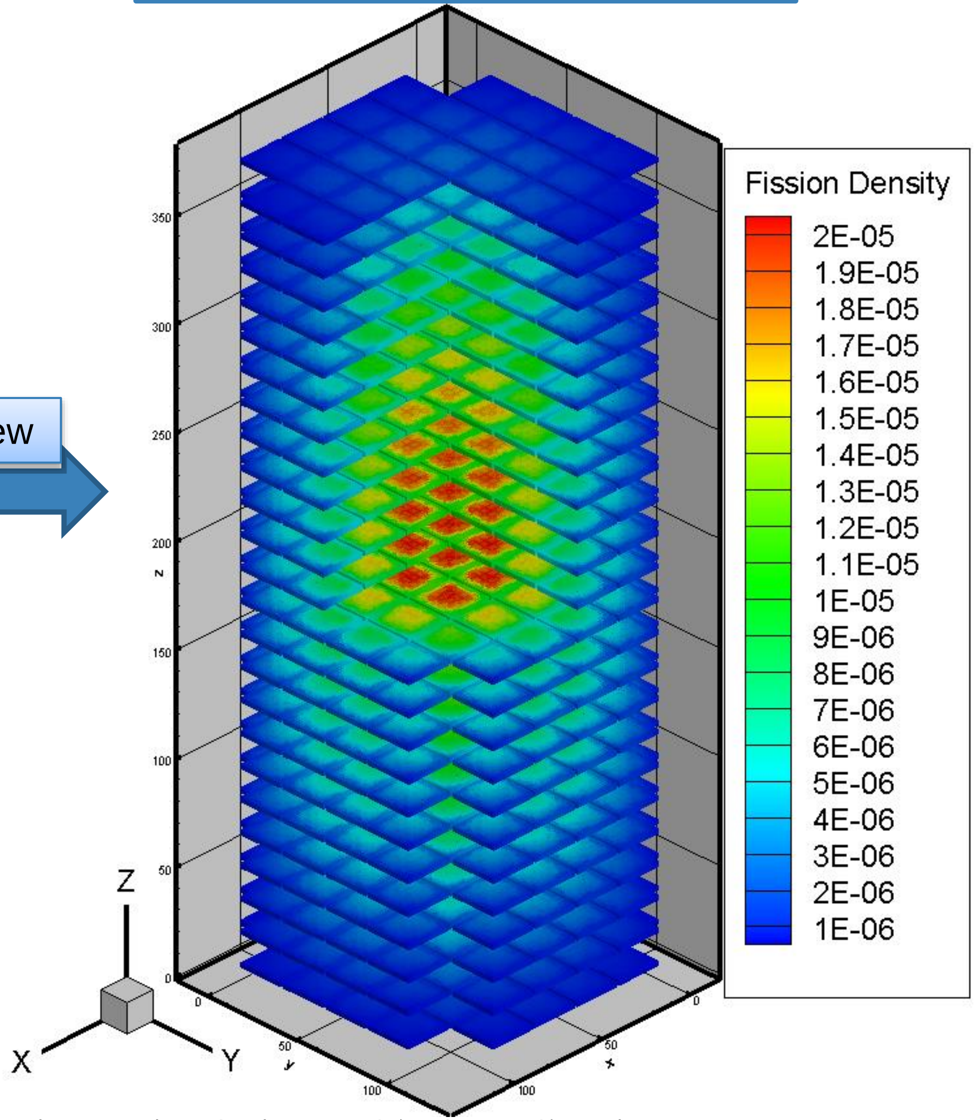
Case	MCNP	RAPID
k_{eff}	1.14545 (± 1 pcm)	1.14590
Relative Difference	-	39 pcm
Fission density rel. uncertainty	1.15%	-
Fission density relative diff.	-	1.56%
Computer	16 cores	1 core
Time	13,767 min (9.5 days)	0.585 min (35 seconds)
Speedup	-	23,533

GBC-32 3D fission density distribution



Inside view

With a quarter Blanked



Concluding remarks and future work

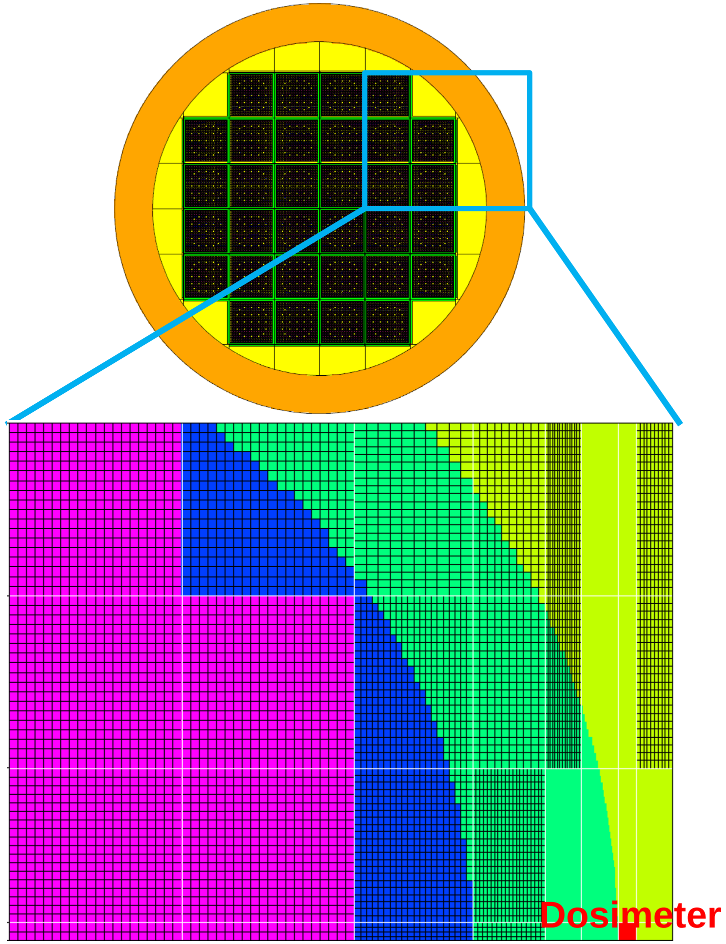
Concluding remarks

▷ It is demonstrated that RAPID can obtain accurate pin-wise, axially-dependent fission source distribution and k_{eff} in a **whole UNF** cask in **real time (seconds)**.

▷ The RAPID MRT algorithm is able to **overcome the main issues** related to Monte Carlo eigenvalue calculations such as source convergence and cycle-to-cycle correlation.

Ongoing & Future work

- External dose/detector response calculation has been implemented in RAPID using the TITAN-calculated importance function methodology*.
- An automated methodology for the determination of the FOV of a detector is under development.
- TITAN dose calculation will be benchmarked against a reference A^3MCNP (Automated Adjoint Accelerated MCNP) code prediction.



* Work presented at the ANTPC conference in Santa Fe, New Mexico, Sep 25-30, 2016.

Questions?

Thanks