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Experimental and Computational Validation of RAPID

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ABSTRACT

The RAPID (Real-time Analysis for Particle transport and In-situ Detection) code system utilizes the Multi-stage Response-function Transport (MRT) approach with the Fission Matrix (FM) method for neutronics simulation of nuclear systems. RAPID performs real-time calculations by utilizing pre-calculated databases for different enrichments, burnups, and cooling times. This paper discusses the validation of RAPID using the U.S. Naval Academy's subcritical reactor (USNA-SCR) facility. Computational validation is performed by detailed comparison with an MCNP reference calculation and experimental validation is performed using both in-core and ex-core neutron measurements with different ^{3}He proportional counters. These measurements, and associated calculations have demonstrated that RAPID achieves accurate results in real-time.

Keywords

RAPID, MRT methodology, Fission Matrix, validation, experimental benchmarking, Monte Carlo

INTRODUCTION

The RAPID Code System [1], developed based on the MRT methodology [2] with the Fission Matrix (FM) and the adjoint function methodologies, is capable of accurately calculating 3-D detailed fission density distribution, subcritical multiplication factor, criticality eigenvalue, and detector response for a nuclear system in real-time. RAPID achieves accurate solutions, comparable to Monte Carlo, while because of its FM method it does not suffer from the eigenvalue Monte Carlo shortcomings including particles under-sampling, source biasing and cycle-to-cycle correlation [3, 4, 5, 6, 7]. Additionally, because of its pre-calculation capability, RAPID can solve complex and large problems in real-time.

Over the past five years, RAPID has been computationally benchmarked against Monte Carlo MCNP [8] calculations of Spent Nuclear Fuel (SNF) pools [9, 10] and casks [3]. In this paper, we present the first experimental benchmark using the U.S. Naval Academy Subcritical Reactor (USNA-SCR) facility [11].

The paper is organized as follows. The RAPID code system is presented, along with the Fission Matrix methodology; the USNA-SCR reactor and the experimental setup is described; RAPID and MCNP results are compared and MCNP tallies are compared to experiments; and, conclusions and remarks are given.

THE RAPID CODE SYSTEM

RAPID is based on the MRT methodology, in which a problem is initially partitioned into a number of stages that are solved independently. These stages are coupled through a linear system of equations with pre-calculated coefficients.

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The Fission Matrix (FM) method

The RAPID code system uses the FM method [4] for the determination of system eigenvalue and fission density distribution. The FM method can be used for both subcritical and critical systems. For a subcritical multiplication problem, in which the fission source is driven by an independent source in the spent fuel (i.e., spontaneous fission and (α, n) reactions), the fission density in cell i is obtained by

$$F_i = \sum_{j=1}^{N}(a_{i,j}F_j + b_{i,j}S_j), \tag{1}$$

where F_j is the induced fission source strength in fuel region j, S_j is the intrinsic (or independent) source strength in fuel region j, $a_{i.j}$ is the number of neutrons directly produced in fuel region i due to a fission neutron born in fuel region j, $b_{i,j}$ is the same as $a_{i,j}$ except for intrinsic source neutrons. These values are different because S and F have different spatial and energy distributions. N is the total number of computational cells.

For the eigenvalue problems the FM formulation is given by

$$F_i = \frac{1}{k}\sum_{j=1}^{N} a_{i,j}F_j, \tag{2}$$

where k is the system eigenvalue, and the remaining terms are same as in Eq. 1.

The above equations can be solved easily via an iterative process. The chief difficulties are: how to calculate the coefficients and, how to decide on the sizes of regions (i's) that yield detailed and accurate results.

To calculate the FM coefficients, a series of fixed-source Monte Carlo MCNP calculations are performed, with the source located in each computational region (i). For this study, a computational region is a segment of a single fuel pin with an axial height of 0.25". The total fission neutron production rate, i.e., $\int dV\ \nu\sigma_f\phi$, is tallied in all surrounding fuel pins for all axial levels (location j), a total of 21,440 tallies regions per calculation. The tally results from all independent fixed-source calculations (268 in total) are compiled to create the FM coefficients, $a_{i,j}'s$. Once the FM coefficients have been calculated, there is no more need for detailed transport calculations.

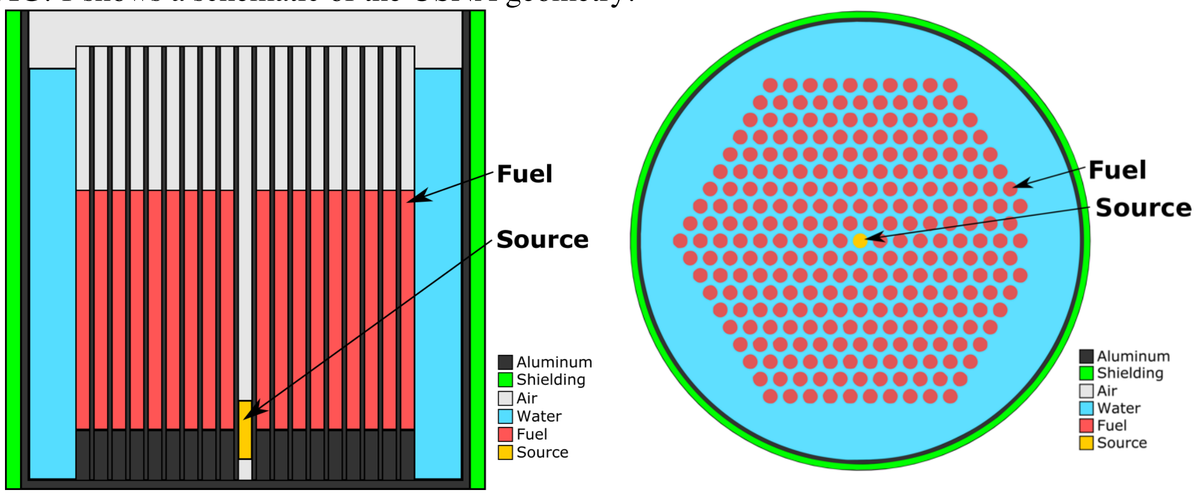
Note that each fixed-source calculation is independent, which allows for these calculations to be performed in parallel. Therefore, the time required is strongly dependent on the computational resources available. For the pre-calculations performed for this study, the time required for one fixed source calculation was between 20 and 30 minutes on a single core. Note that this time requirement could be reduced if multiple cores are used. With our computer cluster consisting of 56 computational cores, the FM coefficient pre-calculations (all $a_{i,j}'s$ and $b_{i,j}'s$) required approximately 250 minutes.

THE USNA-SCR SUBCRITICAL REACTOR

The USNA-SCR is a pool-style subcritical reactor with natural uranium fuel and light water moderator. The pool (see Fig. 1) is an open top, aluminum cylindrical vessel wrapped by borated foam. A total of 268 fuel rods are arranged in a hexagonal pattern, as shown in Fig. 1. Note that there are 3 missing rods on the right-hand side of the fuel bundle (see Fig, 1.a). The fuel rods are constructed of hollow aluminum tubes containing 5 annular fuel slugs which are surrounded by air (they are open at the top). At the bottom of each fuel rod is a hollow acrylic tube which serves to elevate the fuel from the bottom of the pool. The independent neutrons are generated by a plutonium beryllium, PuBe, source. The PuBe material is contained in a stainless steel capsule that is positioned within an aluminum tube in the central location of the fuel assembly. The source capsule is elevated from the bottom of the tube.

Geometry

FIG. 1 shows a schematic of the USNA geometry.



a) Axial view b) Radial view

FIG. 1 Geometry of the USNA-SCR (not drawn to scale)

The pool outer diameter is 121.29 cm with a thickness of 0.64 cm and a height of 152.40 cm. The water depth is 135.66 cm. The borated foam thickness is 0.79 cm. The fuel rod tube outer diameter is 3.45 cm with a thickness of 0.18 cm and a height of 139.70 cm. The fuel rod pitch is 4.81 cm. The fuel slug outer diameter is 3.05 cm with an inner diameter of 1.27 cm and total height of 104.14 cm. The fuel is elevated 15.24 cm from the bottom of the rods. The source capsule outer diameter is 3.02 cm with a total height of 25.30 cm; the PuBe slug (within the capsule) has a diameter of 2.55 cm, a height of 18.50 cm, and is axially centered within the capsule. The source capsule is elevated 5.00 cm from the bottom of the rod.

Neutron Source Material

The source material composition is not explicitly known, but the ratio of plutonium to beryllium is assumed to be 1:13, i.e., $PuBe_{13}$ [12]. Additionally, due to the age of the source, the buildup of Americium as a consequence of the beta decay of ^{241}Pu is considered. The source spectrum has been calculated using the SOURCES-4C [13] code for a material composition obtained from the DOE 2013 Standard [14]. The most probable neutron energy is within 3.00 - 3.25 MeV. The average neutron energy is calculated to be 4.84 MeV and total source strength is 2.32×10^6 n/s/cm^3. FIG.2 shows the SOURCES-4C calculated neutron source spectrum.

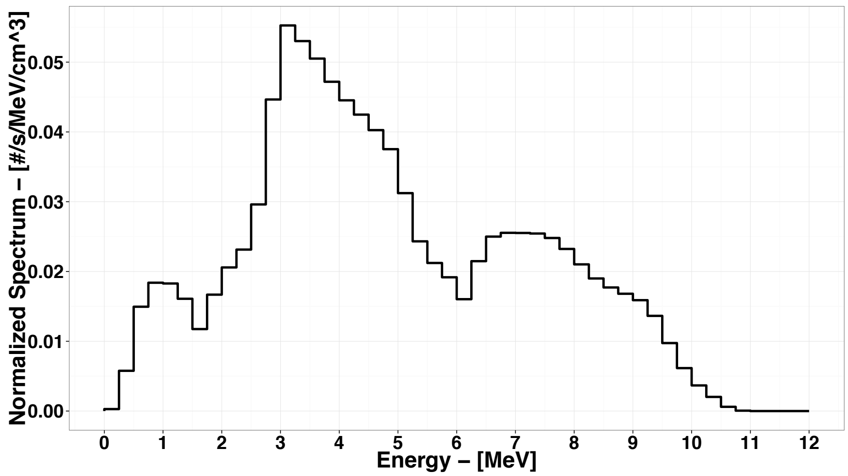


FIG. 2 $PuBe_{13}$ neutron source spectrum calculated using SOURCES-4C [13]

Measurements

Detectors

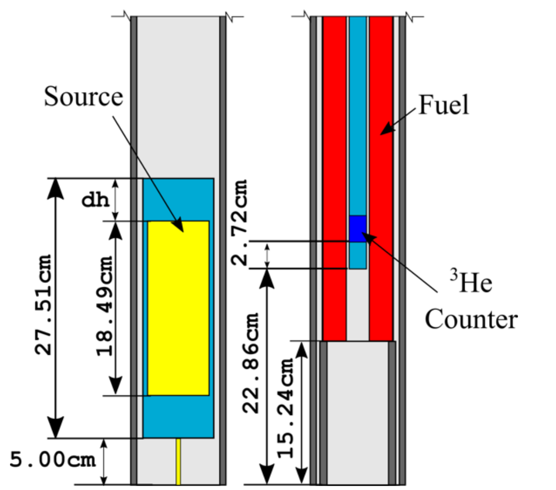
Two detector types were used in order to perform various measurements:

1) Small cylindrical ^3He proportional counter [15] - inserted into the fuel annuli for "in-core" measurements (see FIG. 3a and 3b for dimensions and in-core locations).
2) Large cylindrical ^3He proportional counters [16] - suspended from the top of an empty fuel rod (see FIG. 3c for dimensions and ex-core locations).

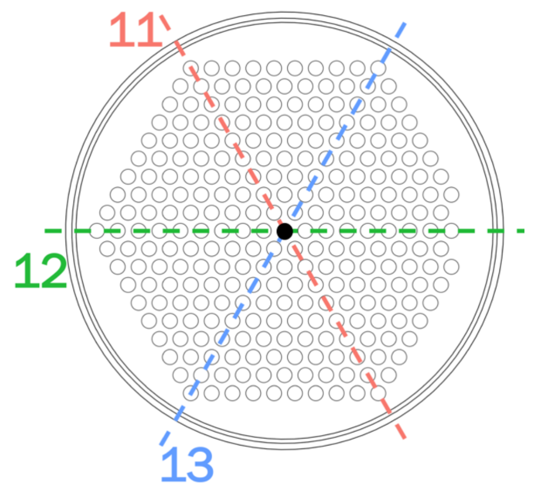
Experimental Setup

Three different sets of measurements have been performed:

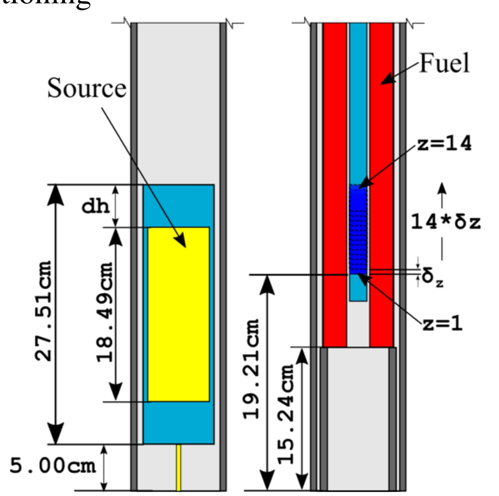
- In-core radial profiles: the small detector was kept at a fixed axial location, elevated 25.58 cm from the bottom of the fuel tubes, and moved along three diagonals (11, 12, and 13) of the hexagonal reactor within the fuel pins. The diagonals used for the radial profiles are depicted in FIG. 3d.
- In-core axial profiles: the small detector was moved axially in steps of 5mm within two selected fuel pins (101, 102) annuli. The fuel pins used for the axial profiles are depicted in FIG. 3e.
- Ex-core measurements: the large detector was positioned at the edges and tips of the fuel bundle. Detector locations for the ex-core measurements are shown in FIG. 3f.



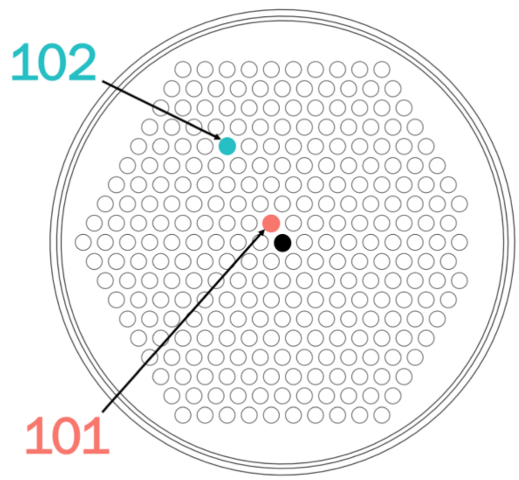
(a) In-Core Radial Profile: Axial detector positioning



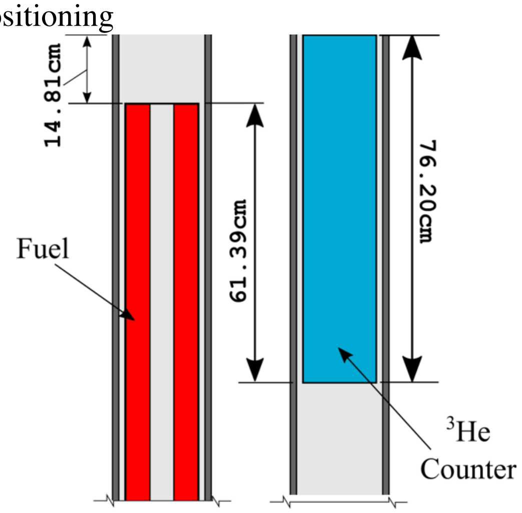
(d) In-Core Radial Profile: Measurement axes



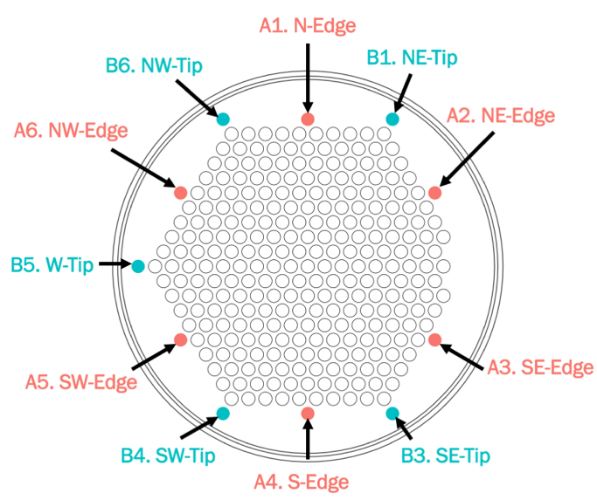
(b) In-Core Axial Profile: Axial detector positioning



(e) In-Core Axial Profile: Fuel pin locations



(c) Ex-Core: Axial detector positioning



(f) Ex-Core: Measurement locations

FIG. 3 Measurement locations and axial detector positioning

RESULTS

In this section, we perform a computational and experimental benchmark for the RAPID Code System using the USNA-SCR facility and the MCNP code.

The source of fission neutrons (i.e., the fission density distribution), the criticality eigenvalue, and the subcritical multiplication factor have been calculated with RAPID and compared to MCNP tallies for the same quantity. The MCNP calculated detector responses are then compared to detector measurements. Note that, for both the RAPID FM coefficient pre-calculations and the MCNP reference calculation the ENDF/B-VII cross-section library was used.

Computational Benchmark

For the reference MCNP eigenvalue calculation the following parameters were used: 10^6 particles per history, 200 skipped cycles, and 300 active cycles. For the reference MCNP subcritical multiplication calculation 10^8 particle histories were tracked. Both the reference calculations included detailed 3D (i.e. pin-wise with 41 axial segments, each 2.54 cm tall) fission density tally bins, $\int_{V_i} \nu\sigma_{f,i}\phi_i$. These calculations resulted in tally-weighted, average uncertainties of 0.28 % for the eigenvalue (*k*) calculation and 0.27 % for the subcritical multiplication (*M*) calculation.

TABLE 1 compares the MCNP and RAPID calculated eigenvalue, *k*, and subcritical multiplication factor, *M*. These values are compared by calculating the relative difference of the RAPID calculated value versus the MCNP reference calculated value. Equation 3 shows the relative difference equation for *k*; note that the relative difference is calculated in the same manner for *M*.

$$Rel.Diff. = \frac{k_{RAPID} - k_{MCNP}}{k_{MCNP}} \tag{3}$$

TABLE 1

Comparison of calculated eigenvalues, *k*, and subcritical multiplication factors, *M*.

Calculation	MCNP	RAPID	Relative Difference
k	0.87278 ± 0.00003	0.87420	163 pcm
M	6.62042 ± 0.00002	6.54819	1.09 %

The above table demonstrates good agreement between the RAPID and the reference MCNP calculations.

TABLE 2 presents a comparison of the required computation time.

TABLE 2

Comparison of required computational resources

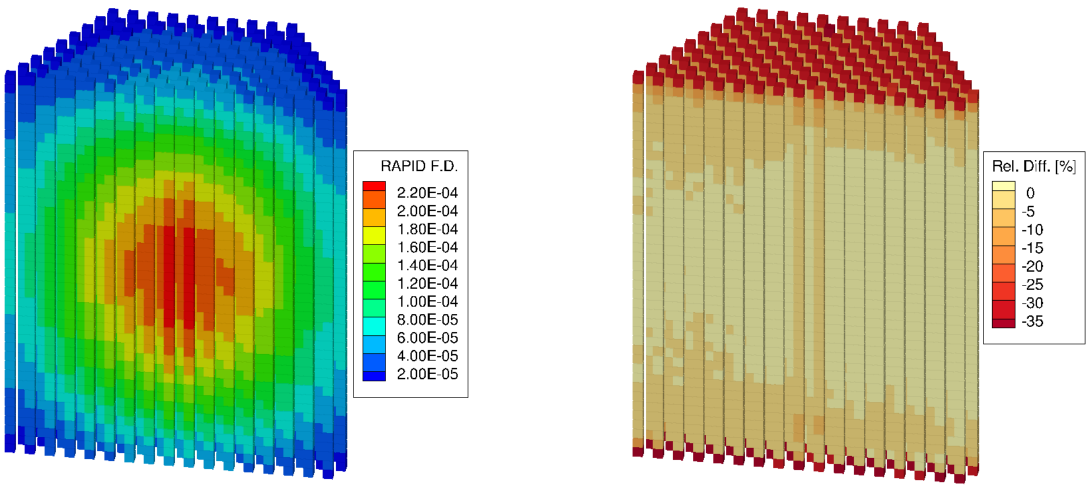
Calculation	MCNP		RAPID		RAPID
	No. Proc.	Time[1] – [min]	No. Proc.	Time – [min]	Speedup
k	16	876	1	0.7	1251

M	16	1134	1	0.8	1418

[1]Wall-clock time.

The above table demonstrates that RAPID performs these calculations in less than one minute on 1 processor (not including the pre-calculation time), while the reference MCNP calculations, performed on 16 processors, require over 800 min.

FIG. 4 shows the RAPID calculated 3D fission density distribution (pin-wise with axial height of 1") from the M calculation, and the relative differences as compared to the MCNP reference calculation.



(a) RAPID calculated fission density distribution (high value in center)

(b) Relative differences of RAPID fission distribution compared to MCNP reference

FIG. 4 RAPID calculated fission density and comparison with MCNP reference calculation

FIG. 4 demonstrates that the RAPID calculated fission density is in good agreement with the MCNP reference calculation, the majority of relative differences are below 5%, and the mean relative difference is approximately 3%. There are noted larger discrepancies on the top and bottom axial levels that is attributed to lower neutron flux and higher statistical uncertainties.

Experimental Benchmark

A fixed-source MCNP calculation was performed, and the ^{3}He reaction rate was tallied at all measurement locations. In order to compare with the measured values, an estimated detector efficiency was calculated, using a least-squares minimization formulation, given by

$$eff = \frac{\sum_i c_i m_i}{\sum_i c_i^2}, \tag{4}$$

where c_i and m_i are the calculated reaction rate and measured reaction rate at location i, respectively. The calculated and experimental reaction rates are compared by calculating the C/E ratios. Additionally, a combined uncertainty of the C/E ratio was calculated as

$$\sigma_{\frac{C}{E},i} = \sqrt{\frac{\sigma_{c,i}^2}{m_i^2} + \frac{c_i^2 \sigma_{m,i}^2}{m_i^4}} \tag{5}$$

where σ_{ci} and σ_{mi} are the uncertainties of the calculated and measured reaction rates, respectively.

FIG. 5 presents the C/E ratios for all the in-core radial profiles measurements along the three diagonals (11, 12, and 13), as depicted in FIG. 3a and FIG. 3d.

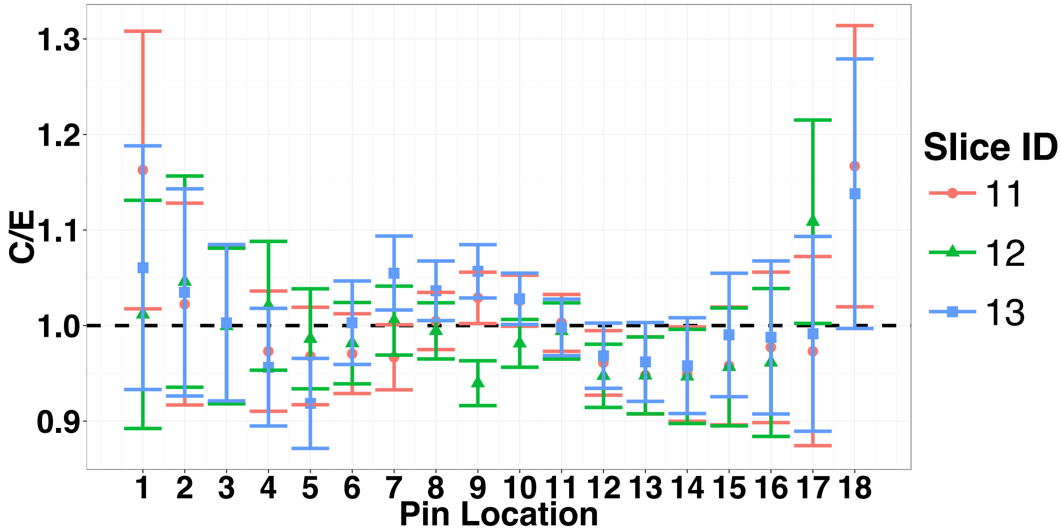


FIG. 5 C/E ratios for the in-core radial profiles

FIG. 5 demonstrates that calculated detector response rates for all in-core radial profiles are in good agreement with the measurements, within +/- 10% (except boundary locations, which exhibit ~5% larger differences). The larger noted differences on the boundary locations are due to the increased measurement uncertainties, caused by the significant reduction of neutron population in the core periphery. The average statistical uncertainty from the MCNP calculation is ~0.5% while the average measurement uncertainty is ~6.0% (this ranges from ~2.5% near the source to ~12.5% at the core periphery)

FIG. 6 presents the C/E ratios for the two in-core axial pin profile measurements, depicted in FIG. 3b and 3e.

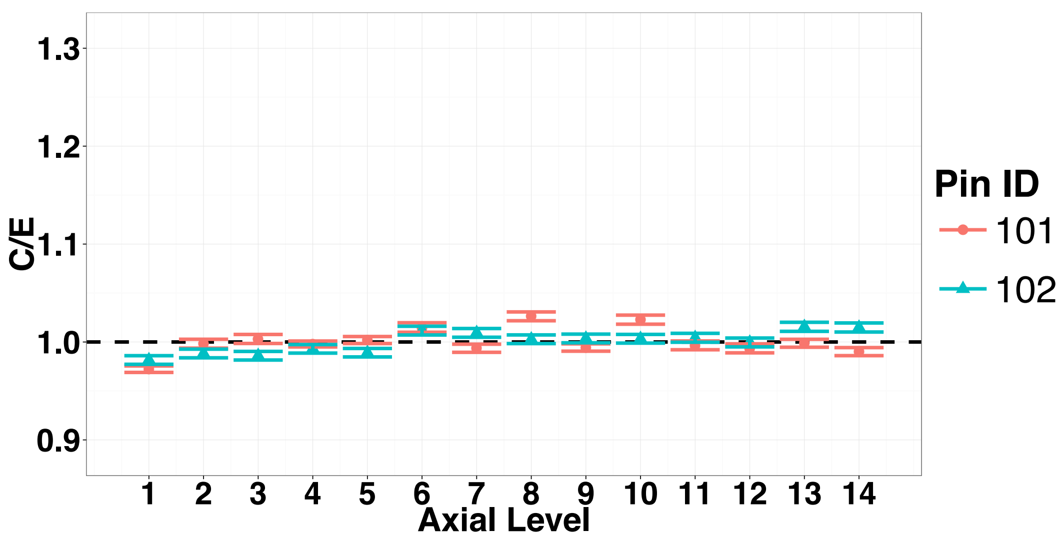


FIG. 6 C/E ratios for the in-core axial profiles

FIG. 6 demonstrates that the calculated detector reaction rates for both in-core axial pin profiles are in very good agreement with the measurements, within ±3%. The average statistical uncertainty on the calculated values is ~0.2% and the average measurement uncertainty is ~0.4%. Note that the error bars on FIG. 6 are significantly smaller than FIG. 5 due to the detector being closer to the source for these axial pin profile measurements.

FIG. 7 presents the C/E ratios for the two sets of ex-core measurements, depicted in FIG. 3c and FIG. 3f.

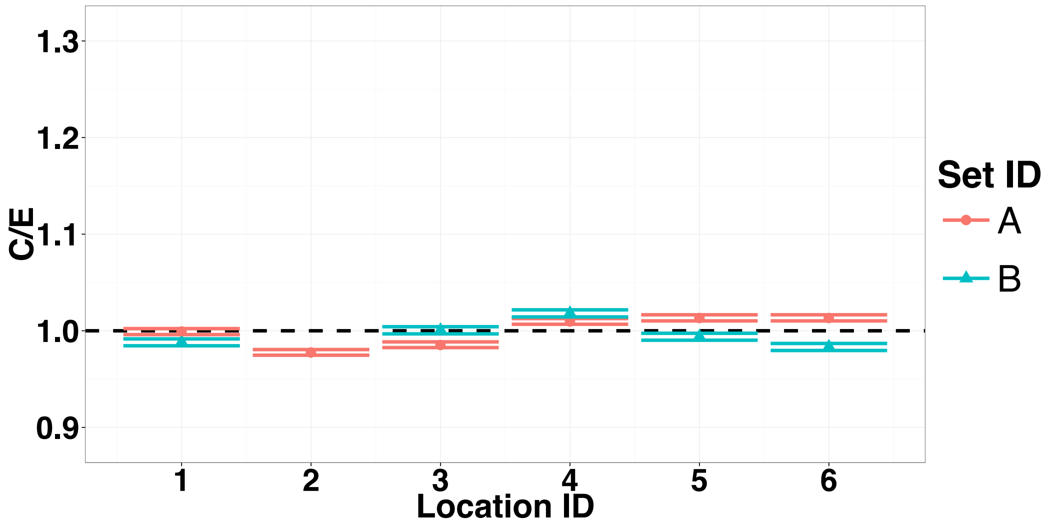


FIG. 7 C/E ratios for all ex-core measurement locations

FIG. 7 demonstrates that the calculated detector reaction rates for both sets of ex-core measurements are in very good agreement with the measurements, within ±3%. The average statistical uncertainty on the calculated values is ~0.3% and the average measurement uncertainty is ~0.2%. Note that the error bars on FIG. 7 are significantly smaller than FIG. 5 due to the increased detector volume used for the ex-core measurements, which significantly decreases the relative measurement uncertainty.

CONCLUSIONS

This paper demonstrates that the RAPID and MCNP calculations are in good agreement, indicating that the RAPID Code System is capable of accurately performing both criticality and subcriticality calculations. In addition to the system eigenvalue, the 3-D fission densities throughout a subcritical facility such as the USNA-SCR are obtained accurately in a fraction of the time with respect to the traditional Monte Carlo calculation.

Our experimental benchmark using the USNA-SCR facility yield good agreement between computed and measured responses for both in-core and ex-core detectors. The observed larger differences can be attributed to the lower neutron populations at the core periphery.

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List of Figure Captions

FIG. 3 RAPID calculated fission density and comparison with MCNP reference calculation

FIG. 4 C/E ratios for the in-core radial core profiles

FIG. 5 C/E ratios for the in-core axial profiles

FIG. 6 C/E ratios for all ex-core measurement locations